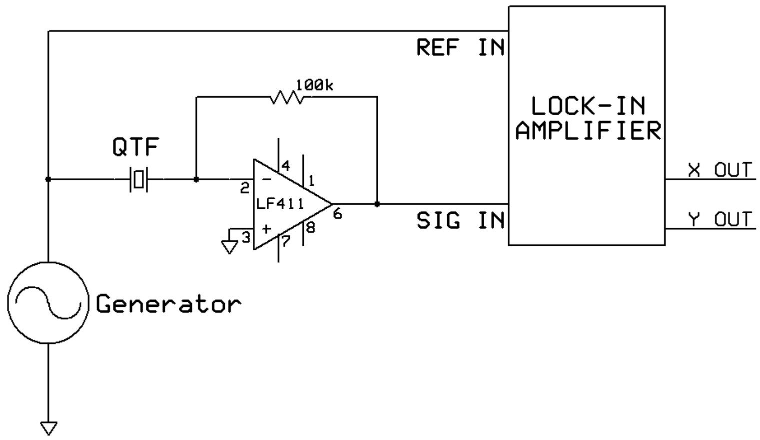
Experiments with Quartz Tuning Forks
Electromechanical Oscillators and Lock-in Amplifiers

Many digital watches use tiny quartz tuning forks as their time-base. The natural frequency of oscillation of the tuning fork provides a stable frequency for measuring time. To both excite and detect these oscillations the piezoelectric nature of quartz is used.

If a block of material is squeezed (subjected to pressure or a stress), it changes shape and dimensions slightly (a strain). In piezoelectric materials, this strain is accompanied by the appearance of a surface electric charge and a electric potential difference between opposite faces of the block. The charge generated is proportional to the amount of strain. The inverse process occurs as well: if a piezoelectric material is subjected to a potential difference or charge is applied to two faces, the material responds by stretching or contracting slightly. For quartz tuning forks, this means that, with clever design of electrical contacts, the arms of the fork can be bent or set into mechanical oscillation, just like a metal tuning fork often found in physics labs, by applying a voltage. The amount of mechanical displacement or bending (x) is directly proportional to the applied voltage or the amount of surface charge, q: $x \propto V \propto q$.

As a mechanical oscillator the tuning fork's motion can be described well as a damped harmonic oscillator. As an electrical device, the tuning fork is modeled as a resonant series R-L-C circuit and is excellent playground for exploring the analogous behavior of resonances in mechanical and electrical systems. In fact, the oscillations of quartz tuning forks are most often described in terms the electrical analog of an R-L-C oscillator. This means that instead of measuring position and velocity of the tuning fork, one measures the charge and current ($x \propto q$, so $i = \frac{dq}{dt} \propto v = \frac{dx}{dt}$).

In this experiment, a function generator will be used to drive the tuning fork electrically with an applied voltage. The motion of the tuning fork is monitored electrically, too, by measuring the resulting electric current, which is proportional to the velocity of the tuning fork arms. When the tuning fork is driven at its resonant frequency ($2^{15} = 32768\,\mathrm{Hz}$ for our tuning forks), there will be a large amplitude oscillating displacement of the arms of the tuning fork, accompanied by a large oscillating surface charge, and a large oscillating electric current proportional to the velocity of the arms. While in a mechanical oscillator we usually think most easily about the displacement as function of time, $x(t)$, in the electrical analog it is easiest to measure the current, $i(t)$, which is proportional to velocity. This current is most easily measured with a simple op-amp in a current-to-voltage converter configuration.



Because this is a damped, driven oscillator, there will be a frequency-dependent phase shift between the driving force (or applied voltage) and the measured response of the oscillator, along with a frequency-dependent amplitude. These two numbers constitute the frequency response of the tuning fork. The goal here will be to measure the frequency response of the tuning fork electrically, determine the resonance frequency and Q (quality factor) by some careful fitting, and explore what happens to the resonant frequency and the Q when the damping is changed by operating the tuning fork in air instead of vacuum. You will get at this information in the form of a measurement of the admittance Y, which is the inverse of impedance, of the tuning fork. Since the impedance of a series RLC circuit is complex:

$$Z = R + i(\omega L - \frac{1}{\omega C}) = |Z|e^{i\phi},$$

the tuning fork current will be

$$i(t) = \frac{v_{in}(t)}{Z} = |Y|e^{-i\phi}v_{in}(t),$$

reflecting a phase shift ϕ from the complex impedance. At resonance the impedance is a minimum and real, so the admittance is a maximum and real. The current will be largest at the resonant frequency and is converted to a voltage by the op-amp with a feedback resistor of R_f (100kΩ is a good starting choice):

$$v_{out} = -i(t)R_f = -|Y|e^{-i\phi}R_f v_{in}(t).$$

Taking the real part and assuming the generator input driving the tuning fork is $v_{in}(t) = V_i\cos(\omega t)$, the op-amp output is $v_{out}(t) = -V_o\cos(\omega t - \phi)$.

To make the amplitude and phase measurements simpler, the response of the tuning fork to the driving voltage will be measured with a two-phase lock-in amplifier, which can extract the both amplitude and phase information simultaneously. Recall that a signal of the form $V_o\cos(\omega t - \phi)$ can be decomposed into

$$V_o\cos(\omega t - \phi) = (V_o\cos\phi)\cos(\omega t) + (V_o\sin\phi)\sin(\omega t)$$

. The SR830 two-phase lock-in carries out this decomposition and provides two outputs simultaneously: the X or in-phase (with reference signal) component, where $X = V_o\cos\phi$, and Y or quadrature component (90 degrees out of phase with the reference) $Y = V_o \sin\phi$. From these two values, either read by a computer-controlled ADC from the BNC outputs or obtained via the GPIB interface, the magnitude $V_o = \sqrt{X^2 + Y^2}$ and phase $\phi = \tan^{-1}(Y/X)$ of the signal into the lock-in can be readily computed and the admittance of the tuning fork found.

A review of oscillators, the analysis of the electrical analog, and applications to tuning forks, including a discussion of the role of the lock-in amplifier is contained in the excerpt from a paper by an M.S. student, K. Billa. Review this first.

Measurements and possible explorations

Preliminary experimentation: Play with the tuning fork and the basic circuit with an oscilloscope before gearing up to use the lock-in amplifier to collect data in LabVIEW. Find roughly the actual resonant frequency for your tuning fork and examine what happens when you change first the amplitude and then the frequency of the driving signal. Look with the scope at both the driving voltage and the response signal from your tuning fork/op-amp circuit. Things to note in doing this: (a) the way both the amplitude of the response and its phase relative to the driving voltage varies around resonance, (b) how long it takes to reach a new steady-state output after a change. A high Q oscillator (little damping) with a sharp resonance can take many cycles to settle into a new state after a change to the driving signal.

Set up to measure in LabVIEW the tuning fork's admittance as a function of frequency with the lock-in amplifier. Produce graphs of the magnitude of Y^2 and the phase ϕ as functions of frequency.

Fit your data to the expected $Y^2(\omega)$ to determine the resonant frequency $f_0 = \omega_0/2\pi$ and Q.

Verify that the response of the tuning fork is linear - doubling the driving voltage doubles the resulting current, within limits. Keep the driving voltage to the tuning fork small: $< 100\,\mathrm{mV}$ amplitude, particularly when the tuning fork is in vacuum in its can; large driving voltages can actually crack or break the tuning fork arms.

Repeat for the tuning fork exposed to air. (Easy way: File a hole in the can, harder: crack the tuning fork loose from the can by gently squeezing the base of the can repeatedly until the glass seal in the bottom disintegrates and the fork and leads are free of the can. This requires some practice.) In air, the tuning fork peak in the resonance curve changes compared to its vacuum value, both decreasing in amplitude and shifting the peak location. Is the peak in response shifted because the increased damping and therefore Q is different, or for some other reason? What else might cause this change?

Miscellaneous notes:

The lock-in has a lot of controls and options that are overwhelming to the beginner. The most important controls include:
the **sensitivity**, which sets the largest signal the lock-in can process (it's basically a range setting and sets gain on internal amplifier stages),
the **time constant of the low pass output filter**, which filters out remaining noise from the mixer; noisy signals require longer time constants to narrow the bandwidth of the measurement, and a simple rule of thumb is that any change in the experiment (drive level or frequency, for example) requires at least a 3 time-constant wait before the lock-in outputs have settled to their new values (assuming the tuning fork itself settles quickly - that's not always the case!);
setting the **reference** settings to external, with no additional phase shifts added to the reference signal, and connecting the driving generator to the reference input;
making sure the **outputs** are set for the desired quantities.