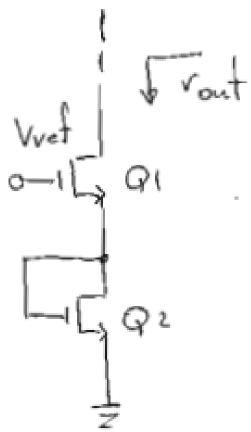
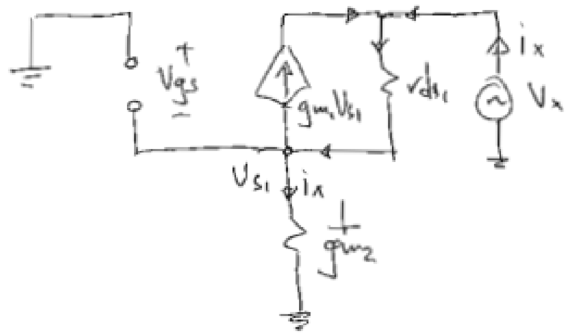
3.19



Ignoring the body effect,

the small-signal model is



KCL @ V_x: $\quad g_{m1} V_{s1} + i_x = (V_x - V_{s1})\, g_{ds1}$ $\qquad (1)$

KVL @ V_{s1}: $\quad V_{s1} = i_x \dfrac{1}{g_{m2}}$ $\qquad (2)$

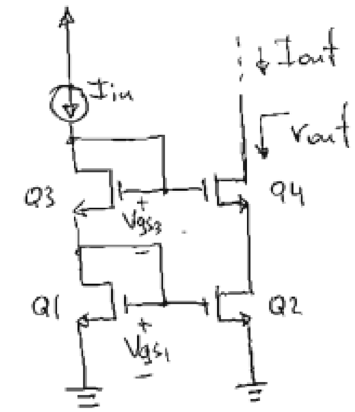
$(1) \Rightarrow \quad V_{s1}(g_{m1} + g_{ds1}) + i_x = V_x g_{ds1}$

Sub-in (2) $\Rightarrow \quad i_x \left[\dfrac{1}{g_{m2}}(g_{m1} + g_{ds1}) + 1 \right] = V_x g_{ds1}$

$$r_{out} = \frac{V_x}{i_x} = \frac{1 + \frac{1}{g_{m2}}(g_{m1} + g_{ds1})}{g_{ds1}} = r_{ds1}\left(1 + \frac{g_{m1} + g_{ds1}}{g_{m2}}\right)$$

$$\approx r_{ds1}\left(1 + \frac{g_{m1}}{g_{m2}}\right)$$

3.20



0.35 μm CMOS:

$\mu_n C_{ox} = 190 \frac{\mu A}{V^2}$

$V_{tn} = 0.57V$

$$V_{d3} = V_{gs1} + V_{gs3}$$

$$= (V_{eff1} + V_{tn1}) + (V_{eff3} + V_{tn3})$$

$$= 2V_{eff_n} + 2V_{tn}$$

For Q4 to be in saturation:

$$V_{gd4} \le V_{tn}$$

$$V_{d3} - V_{out} \le V_{tn}$$

$$V_{out} \ge V_{d3} - V_{tn}$$

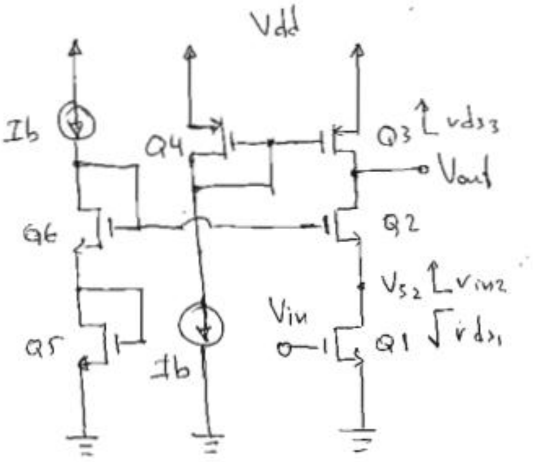
$$V_{out,min} = V_{d3} - V_{tn}$$

$$= 2V_{eff_n} + V_{tn}$$

$$V_{eff} = \sqrt{\frac{2 i_d}{\mu_n C_{ox} \left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.3mA)}{\left(0.19\frac{mA}{V^2}\right)\left(\frac{5\mu m}{0.5\mu m}\right)}} = 178mV$$

$$V_{out,min} = 2(178mV) + 570mV = 0.93V$$

3.23



0.35 μm CMOS:

$\mu_n C_{ox} = 190 \frac{\mu A}{V^2}$

$\mu_p C_{ox} = 55 \frac{\mu A}{V^2}$

$V_{tn} = 0.57V$

$V_{tp} = -0.71V$

$\lambda_n L = 0.16 \frac{\mu m}{V}$

$\lambda_p L = 0.16 \frac{\mu m}{V}$

a) $i_{d1} = i_{d3} = \frac{(W/L)_3}{(W/L)_4} i_{bias} = i_{bias}$

$$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) i_{bias}}$$

$$= \sqrt{2\left(190 \frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{0.5\mu m}\right) 100\mu A}$$

$$= 0.872 \frac{mA}{V}$$

$$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right) i_{bias}}$$

$$= \sqrt{2\left(55 \frac{\mu A}{V^2}\right)\left(\frac{30\mu m}{0.5\mu m}\right) 100\mu A}$$

$$= 0.812 \frac{mA}{V}$$

$$V_{eff_{1,2}} = \sqrt{\frac{2 I_{d1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2\,(100\mu A)}{\left(190 \frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{0.5\mu m}\right)}} = 0.23V$$

$$V_{eff_3} = \sqrt{\frac{2 i_{d1}}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(100\mu A)}{\left(55 \frac{\mu A}{V^2}\right)\left(\frac{30\mu m}{0.5\mu m}\right)}} = 0.25V$$

$$r_{ds1} = r_{ds2} = r_{ds3} = \frac{L}{(\lambda_n L)(i_b)}$$

$$= \frac{0.5\mu m}{\left(0.16 \frac{\mu m}{V}\right)(100\mu A)} = 31.25 k\Omega$$

$$A_v = \frac{V_{s2}}{V_{in}} \cdot \frac{V_{out}}{V_{s2}} \;; \quad \frac{V_{s2}}{V_{in}} = -g_{m1}\left(r_{ds1} \| r_{in2}\right)$$

$$r_{in2} = \frac{1}{g_{m2}}\left(1 + \frac{r_{ds3}}{r_{ds2}}\right) \approx \frac{2}{g_{m2}} \;; \quad \frac{V_{out}}{V_{s2}} = g_{m2}\left(r_{ds2} \| r_{ds3}\right)$$

$$A_v = -g_{m1}\left(r_{ds1} \| \frac{2}{g_{m2}}\right) \cdot g_{m2}\left(r_{ds2} \| r_{ds3}\right)$$

$$\approx -g_{m1} \frac{2}{g_{m2}} \cdot g_{m2} \frac{1}{2} r_{ds2} = -g_{m1} r_{ds2}$$

$$= -\left(0.872 \frac{mA}{V}\right)(31.25 k\Omega) = -27.3 \text{ V/V}$$

3.23

b) $P_{tot} = V_{dd}\left(i_{d6} + i_{d4} + i_{d3}\right) = 3.3V\left(100\mu A + 100\mu A + 100\mu A\right)$

$= 0.99\ mW$

c) $V_{out,min} = V_{gs5} + V_{gs6} - V_{tn}$

$= 2V_{eff} + V_{tn}$

$= 2(0.23V) + 0.57V$

$= 1.03V$

Q3 SAT: $V_{gd3} \geq V_{tp}$

$V_{dd} - V_{sg3} - V_{out} \geq V_{tp}$

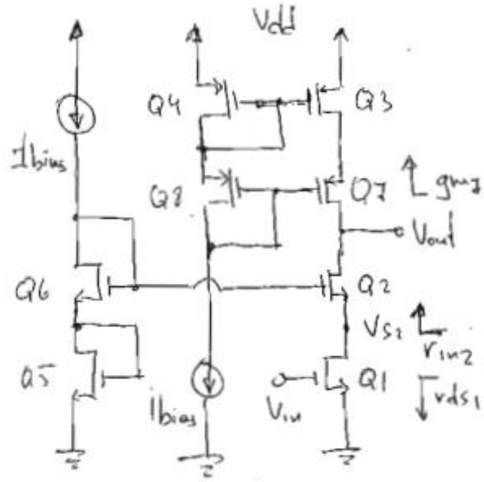
$V_{out} \leq V_{dd} - \left(|V_{eff3}| + |V_{tp}|\right) - V_{tp}$

$= 3.3V - \left(0.25V + 0.71V\right) - \left(-0.71V\right)$

$= 3.05V$

$V_{out,max} = 3.05V$

3.25



0.35 μm CMOS:

$\mu_n C_{ox} = 190 \frac{\mu A}{V^2}$

$\mu_p C_{ox} = 55 \frac{\mu A}{V^2}$

$V_{tn} = 0.57\,V$

$V_{tp} = -0.71\,V$

$\lambda_n L = 0.16 \frac{\mu m}{V}$

$\lambda_p L = 0.16 \frac{\mu m}{V}$

a) $i_{d1} = i_{d3} = \frac{(W/L)_3}{(W/L)_4} i_{bias} = i_{bias}$

$g_{m1} = g_{m2} = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right) i_{bias}}$

$= \sqrt{2 \left(190 \frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{0.5\mu m}\right) 100\mu A}$

$= 0.872 \frac{mA}{V}$

$g_{m3} = g_{m7} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right) i_{bias}}$

$= \sqrt{2 \left(55 \frac{\mu A}{V^2}\right)\left(\frac{30\mu m}{0.5\mu m}\right)(100\mu A)}$

$= 0.812 \frac{mA}{V}$

$V_{eff_{1,2}} = \sqrt{\frac{2 I_{d1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(100\mu A)}{\left(190 \frac{\mu A}{V^2}\right)\left(\frac{10\mu m}{0.5\mu m}\right)}} \approx 0.23\,V$

$V_{eff_{3,7}} = \sqrt{\frac{2 i_{d1}}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(100\mu A)}{\left(55 \frac{\mu A}{V^2}\right)\left(\frac{30\mu m}{0.5\mu m}\right)}} = 0.25\,V$

$r_{ds_{1,2,3,7}} = \frac{L}{\lambda_n L\, i_d} = \frac{L}{\lambda_p L\, i_d} = \frac{0.5\,\mu m}{\left(0.16 \frac{\mu m}{V}\right)(100\mu A)}$

$= 31.25\,k\Omega$

$A_v = \frac{V_{s2}}{V_{in}} \cdot \frac{V_{out}}{V_{s2}}$; $\frac{V_{s2}}{V_{in}} = -g_{m1}(r_{ds1} \| r_{in2})$

$g_{in2} = \frac{1}{r_{in2}} = \frac{g_{m2}}{1 + \frac{R_L}{r_{ds2}}}$, $R_L = g_{m7} r_{ds3} r_{ds7} = g_{m7} r_{ds}^2$

$g_{in2} = \frac{g_{m2}}{1 + g_{m7} r_{ds}} = \frac{0.872 \frac{mA}{V}}{1 + 0.812 \frac{mA}{V}\, 31.25 k\Omega} = 0.0331 \frac{mA}{V}$

$r_{in2} = \frac{1}{g_{in2}} = 30.25\,k\Omega$

$\frac{V_{s2}}{V_{in}} = -\left(0.872 \frac{mA}{V}\right)\left(31.25 k\Omega \| 30.25 k\Omega\right) = -13.4\ V/V$

3.25

$$\frac{V_{out}}{V_{s2}} = g_{m2}(r_{ds2} \parallel R_L) \quad ; \quad R_L = g_{m7} r_{ds3} r_{ds7}$$

$$= g_{m7} r_{ds}^2$$

$$\approx g_{m2} r_{ds2}$$

$$= \left(0.872 \frac{mA}{V}\right) \cdot (31.25 k\Omega)$$

$$= 27.25 \text{ V/V}.$$

$$A_v = \frac{V_{s2}}{V_{in}} \cdot \frac{V_{out}}{V_{s2}} = \left(-13.4 \frac{V}{V}\right)\left(27.25 \text{ V/V}\right) = -365.2 \text{ V/V}$$

$$r_{out} = R_L \parallel r_{d2} = g_{m7} r_{ds}^2 \parallel g_{m2} r_{ds}^2$$

$$\approx \frac{1}{2} g_{m7} r_{ds}^2 = \frac{1}{2}\left(0.812 \frac{mA}{V}\right)(31.25 k\Omega)^2$$

$$= 397 k\Omega$$

b) $P_{TOT} = V_{dd} \times (i_{d6} + i_{d4} + i_{d3})$

$$= 3.3V\,(100\mu A \times 3)$$

$$= 0.99 \, mW$$

3.25

c) $V_{out,min} = V_{gs5} + V_{gs6} - V_{tn2}$

$= 2V_{eff_n} + V_{tn}$

$= 2(0.23V) + 0.57V$

$\approx 1.03V$

$V_{out,max} \approx V_{dd} - V_{sg4} - V_{sg8} + |V_{tp}|$

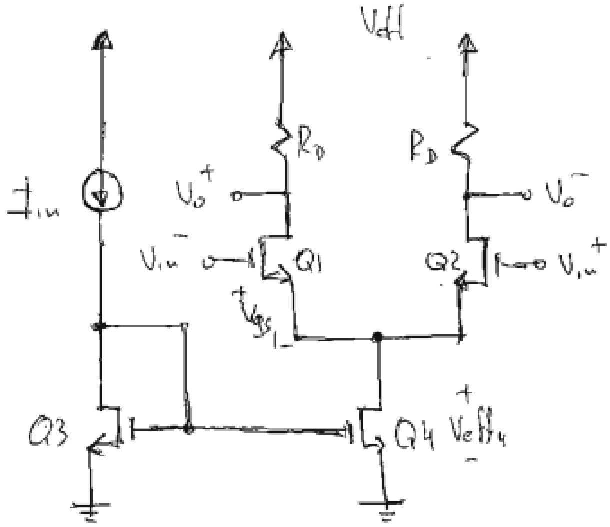
$= V_{dd} - 2|V_{eff_p}| - 2|V_{tp}| + |V_{tp}|$

$\approx 3.3V - 2(0.25V) - 0.71V$

$\approx 2.09V$

d) The addition of Q7 and Q8 increases the gain but lowers the output swing.

3.30



0.18μm CMOS:

$\mu_n C_{ox} = 270\ \frac{\mu A}{V^2}$

$\mu_p C_{ox} = 70\ \frac{\mu A}{V^2}$

$\lambda_n L = 0.08\ \frac{\mu m}{V}$

$\lambda_p L = 0.08\ \frac{\mu m}{V}$

$V_{tn} = 0.45V$

$V_{tp} = -0.45V$

$$V_{gs_{1,2}} = V_{eff_1} + V_{tn}$$

$$= 0.3V + 0.45V$$

$$= 0.75V$$

$$V_{eff_4} = V_{CM,MIN} - V_{gs_{1,2}}$$

$$= 1V - 0.75V$$

$$= 0.25V$$

$$W_4 = \frac{2 i_{d4} L}{\mu_n C_{ox} V_{eff_4}^2}$$

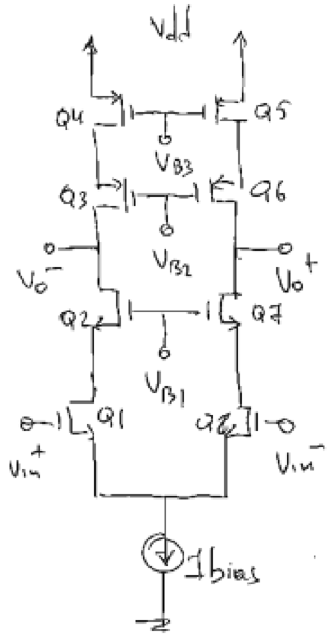
$$= \frac{2 \times 556\mu A \cdot 0.25\mu m}{270 \frac{\mu A}{V^2} \cdot (0.25V)^2}$$

$$= 16.5\mu m$$

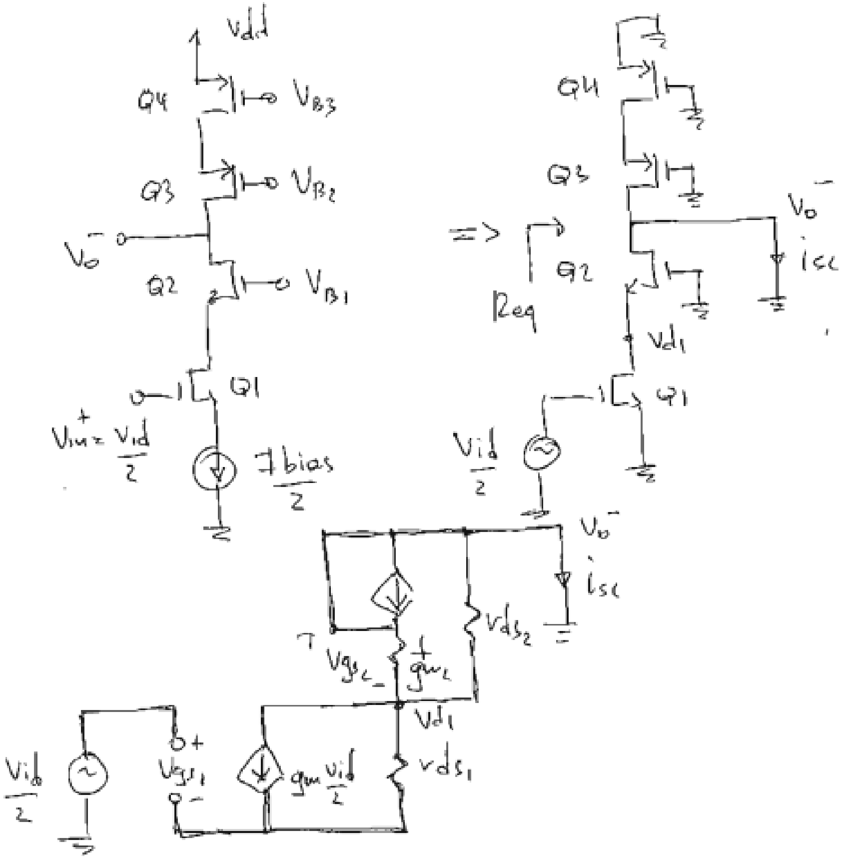
$$\frac{(W/L)_4}{(W/L)_3} = \frac{555.6\mu A}{25\mu A}$$

$$W_3 = \frac{25}{555.6} W_4 = 0.74\mu m$$

3.32



a) Equivalent half-circuit:



b) $$i_{sc} = -g_{m1}\frac{V_{id}}{2}\,\frac{r_{ds1}}{r_{ds1}+\frac{1}{g_{m2}}\| r_{ds2}} \approx -g_{m1}\frac{V_{id}}{2}\,\frac{r_{ds1}}{r_{ds1}+\frac{1}{g_{m2}}} \approx -g_{m1}\frac{V_{id}}{2}$$

$$R_{eq} \approx r_{ds3}\,g_{m3}\,r_{ds4} \,\|\, r_{ds2}\,g_{m2}\,r_{ds1}$$

$$V_o^- = -\frac{1}{2}g_{m1}V_{id}\left(g_{m3}r_{ds3}r_{ds4} \,\|\, g_{m2}r_{ds2}r_{ds1}\right)$$

$$\frac{V_o}{V_{id}} = \frac{V_o^+ - V_o^-}{V_{id}} = g_{m1}\left(g_{m3}r_{ds3}r_{ds4} \,\|\, g_{m2}r_{ds2}r_{ds1}\right)$$

Note: for this question, you can treat it as a cascade amplifier and apply the results for the cascade amplifier as in Question 3.25.