Noise and linearity analysis and modeling

Chapter 9

To develop good analog circuit design techniques, a basic understanding of noise is required. Noise is especially important in OpAmps, filters and converters.

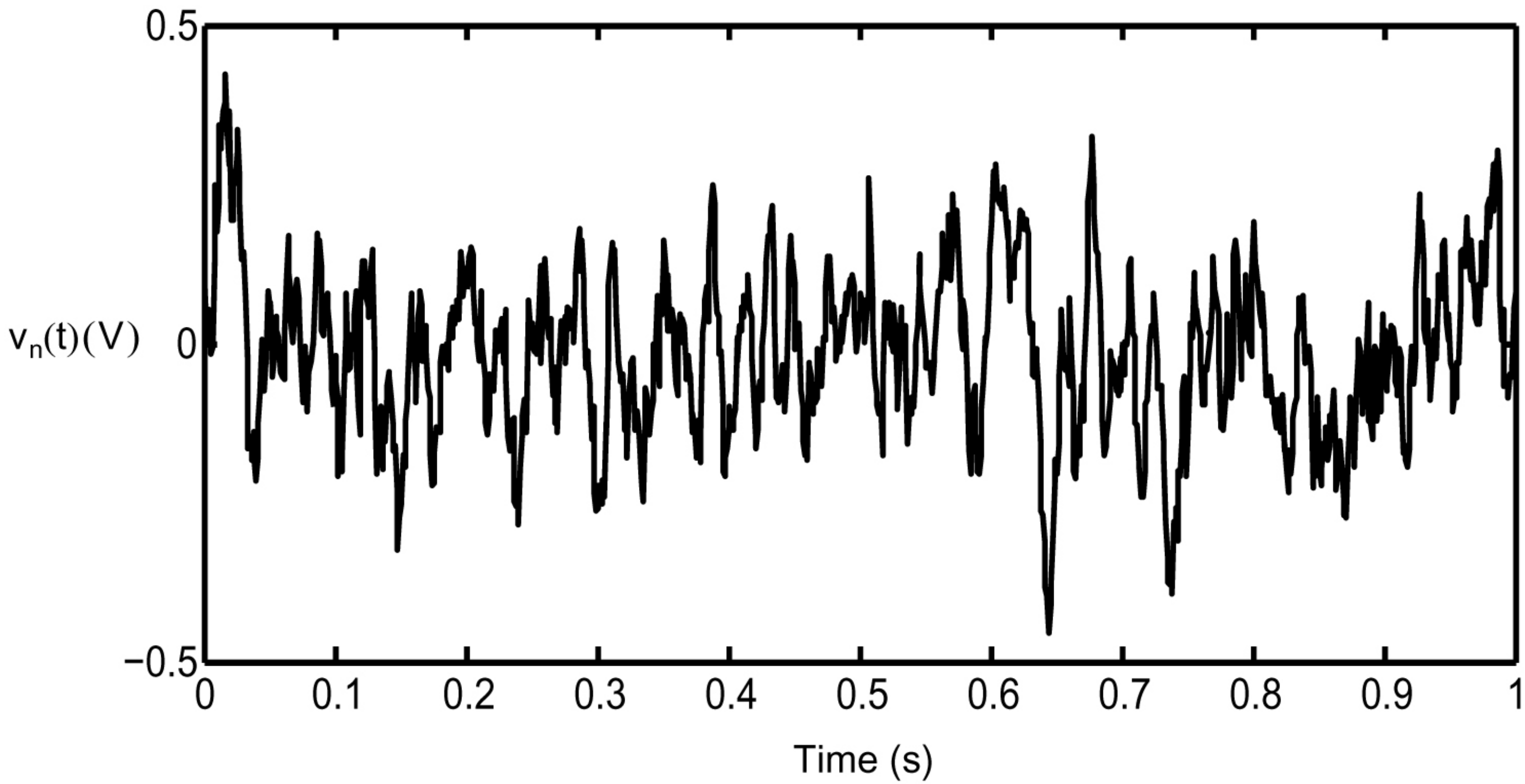
In this Chapter, we focus one inherent noise as opposed to interference noise. The later is a result of unwanted interaction between the circuit and the outside world or different parts of the circuit. Interference noise may or may appear as random signals, such as power supply noise or coupling noise between wires, but usually they can be significantly reduced by careful wiring and layout.

In contrast, inherent noise refers to random noise signals that can be reduced but never eliminated since this noise is due to fundamental properties of the circuits. It is only moderately affect by circuit wiring and layout. However, inherent noise can significantly reduced through proper circuit design, such as sizes, power, circuit topology etc.

9.1 Time-domain analysis

Since inherent noise is random in nature, we can use statistics to deal with random signals.

Throughout this Chapter, we will assume that all noise signals have a mean value of 0, which is valid in most physical systems.



Chapter 9 Figure 01

9.1.1 RMS value

Consider a noise voltage, $v_n(t)$, such as that shown in Fig. 9.1, or a noise current, $i_n(t)$. The *rms*, or *root mean square*, voltage value is defined[1] as

$$V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)\,dt\right]^{1/2} \tag{9.1}$$

where T is a suitable averaging time interval. Typically, a longer T gives a more accurate rms measurement. Similarly, the rms current value is defined as

$$I_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T i_n^2(t)\,dt\right]^{1/2} \tag{9.2}$$

The benefit in knowing the rms value of a signal is that it indicates the *normalized noise power* of the signal. Specifically, if the random signal $v_n(t)$ is applied to a $1\text{-}\Omega$ resistor, the average power dissipated, P_{diss}, in watts, equals the normalized noise power and is given by

$$P_{diss} = \frac{V_{n(rms)}^2}{1\ \Omega} = V_{n(rms)}^2 \tag{9.3}$$

9.1.2 SNR

The *signal-to-noise ratio (SNR)* value (in dB) of a signal node in a system is defined as

$$\text{SNR} \equiv 10\log\left[\frac{\text{signal power}}{\text{noise power}}\right]\text{dB}$$

Thus, assuming a node in a circuit consists of a signal, $v_x(t)$, that has a normalized signal power of $V^2_{x(rms)}$ and a normalized noise power of $V^2_{n(rms)}$, the SNR is given by

$$\text{SNR} = 10\log\left[\frac{V^2_{x(rms)}}{V^2_{n(rms)}}\right] = 20\log\left[\frac{V_{x(rms)}}{V_{n(rms)}}\right]\text{dB} \qquad (9.6)$$

Clearly, when the mean-squared values of the noise and signal are the same, then $\text{SNR} = 0$ dB.

9.1.3 Units of dBm

Although dB units relate the relative ratio of two power levels, it is often useful to know a signal's power in dB on an absolute scale. One common measure is that of dBm, where all power levels are referenced by 1 mW. In other words, a 1-mW signal corresponds to 0 dBm, whereas a 1-μW signal corresponds to -30 dBm. When voltage levels are measured, it is also common to reference the voltage level to the equivalent power dissipated if the voltage is applied across either a 50-Ω or a 75-Ω resistor.

EXAMPLE 9.1

Find the rms voltage of a 0-dBm signal referenced to a 50-Ω resistor. What is the level in dBm of a 2-volt rms signal?

Solution

A 0-dBm signal referenced to a 50-Ω resistor implies that the rms voltage level equals

$$V_{(rms)} = \sqrt{(50\ \Omega) \times 1\ \text{mW}} = 0.2236\ \text{V} \tag{9.7}$$

Thus, a 2-volt rms signal corresponds to

$$20\log\left(\frac{2.0}{0.2236}\right) = 19\ \text{dBm} \tag{9.8}$$

and would dissipate 80 mW across a 50-Ω resistor.

9.1.4 Noise summation

Define $v_{no}(t)$ as $\quad v_{no}(t) = v_{n1}(t) + v_{n2}(t)$

where $v_{n1}(t)$ and $v_{n2}(t)$ are two noise sources with known rms values $V_{n1(rms)}$ and $V_{n2(rms)}$, respectively.

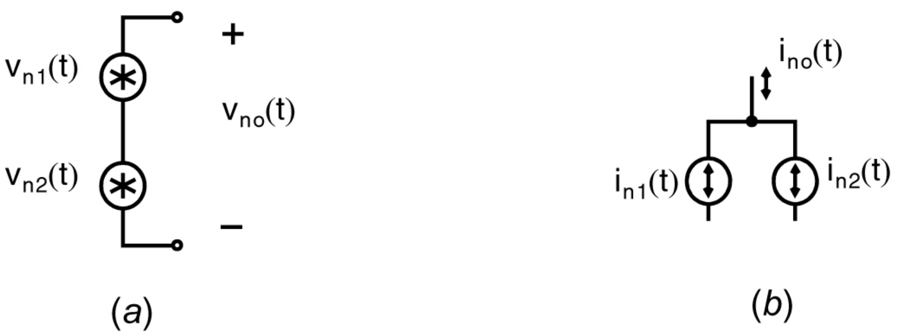
$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\, dt = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt$$

define a correlation coefficient, C, as $\quad C \equiv \dfrac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt}{V_{n1(rms)}V_{n2(rms)}} \qquad -1 \le C \le 1.$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + 2CV_{n1(rms)}V_{n2(rms)}$$

In the case of two uncorrelated signals, the mean-squared value of their sum is given by

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)}$$



Chapter 9 Figure 02

9.1.4 Noise summation

Define $v_{no}(t)$ as $v_{no}(t) = v_{n1}(t) + v_{n2}(t)$

where $v_{n1}(t)$ and $v_{n2}(t)$ are two noise sources with known rms values $V_{n1(rms)}$ and $V_{n2(rms)}$, respectively.

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\,dt = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)\,dt$$

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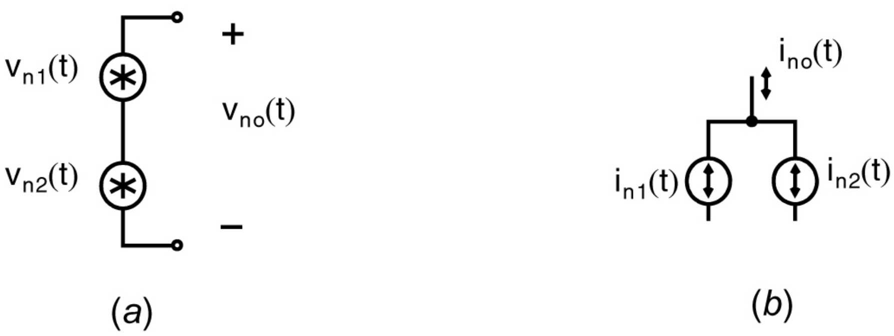


Chapter 9 Figure 02

EXAMPLE 9.2

Given two uncorrelated noise sources that have $V_{n1(rms)} = 10\ \mu V$ and $V_{n2(rms)} = 5\ \mu V$, find their total output rms value when combined. If we are required to maintain the total rms value at $10\ \mu V$, how much should $V_{n1(rms)}$ be reduced while $V_{n2(rms)}$ remains unchanged?

Solution

Using (9.14) results in

$$V_{no(rms)}^2 = (10^2 + 5^2) = 125(\mu V)^2 \tag{9.16}$$

which results in $V_{no(rms)} = 11.2\ \mu V$.

To maintain $V_{no(rms)} = 10\ \mu V$ and $V_{n2(rms)} = 5\ \mu V$, we have

$$10^2 = V_{n1(rms)}^2 + 5^2 \tag{9.17}$$

which results in $V_{n1(rms)} = 8.7\ \mu V$. Therefore, reducing $V_{n1(rms)}$ by 13 percent is equivalent to eliminating $V_{n2(rms)}$ altogether!

The above example has an important moral. *To reduce overall noise, concentrate on large noise signals*.

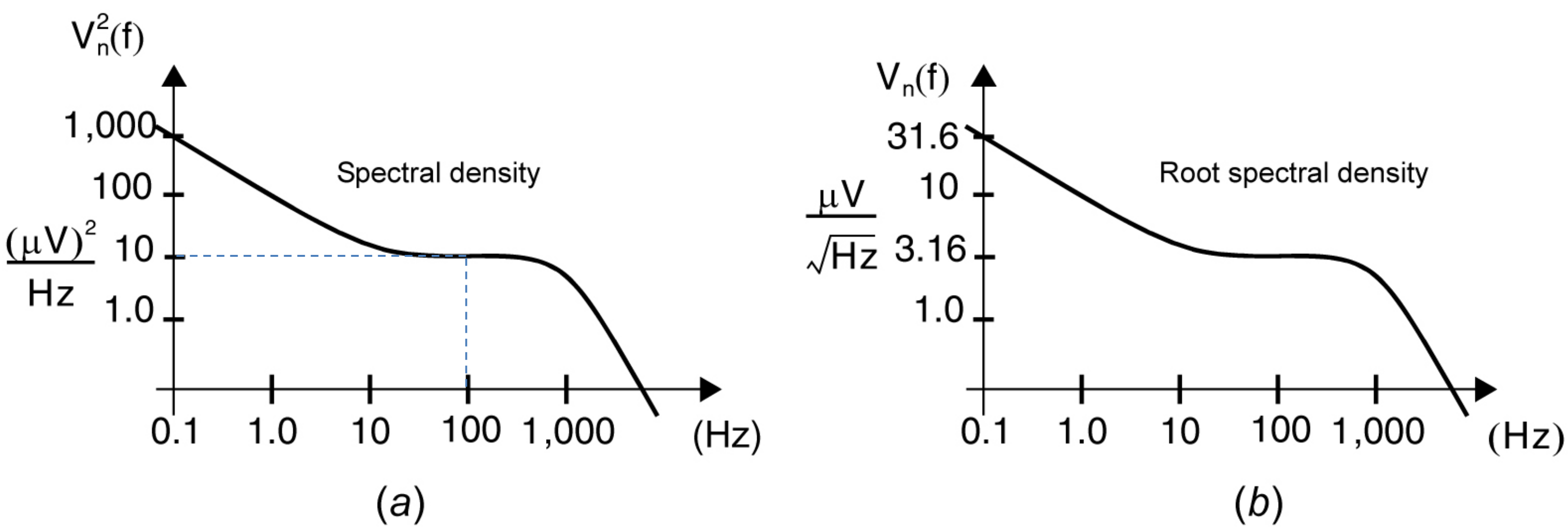
9.2 Frequency domain analysis

As with deterministic signals, frequency-domain techniques are useful for dealing with random signals such as noise.

9.2.1 Noise spectral density

Although periodic signals (such as a sinusoid) have power at distinct frequency locations, random signals have their power spread out over the frequency spectrum. A noise spectral plot is useful to understand the power distribution of the noise signal.

In the plot (a) below, the vertical axis is a measure of the normalized noise power (mean squared value) over a 1-Hz bandwidth at each frequency point. For example, the measurement at 100Hz indicates the normalized power between 99.5Hz and 100.5Hz is $10\ (\mu V)^2$.

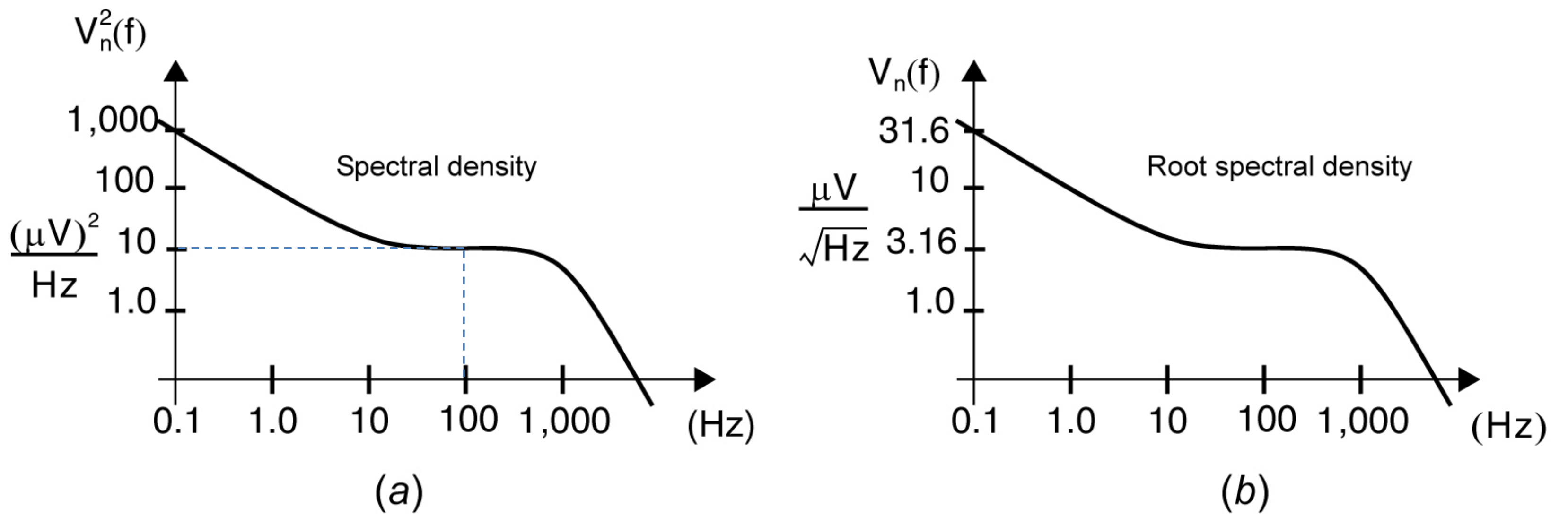


Chapter 9 Figure 03

9.2.1 Noise spectral density

Thus, we define the noise spectral density, $V_n^2(f)$, or in the case of current, $I_n^2(f)$, as the average normalized noise power over a 1-Hz bandwidth. The units of $V_n^2(f)$ are volts-squared/hertz, whereas those of $I_n^2(f)$ are amps-squared/hertz. Also, $V_n^2(f)$ is a positive real-valued function.

It is often convenient to plot the square root of the noise spectral density when we deal with filtered noise. Taking a square root results in $V_n(f)$, as shown in Fig. 9.3(*b*). We will refer to $V_n(f)$ as the *root spectral density which is expressed in units of volts/root-hertz* (i.e., $V/\sqrt{Hz}$). In the case of current noise, the resulting units are *amps/root-hertz*. Note that the horizontal axis remains unchanged although there is a root-hertz factor in the vertical axis.



Chapter 9 Figure 03

9.2.1 Noise spectral density

Since the spectral density measures the mean-squared value over a 1-Hz bandwidth, one can obtain the total mean-squared value by integrating the spectral density over the entire frequency spectrum. Thus, the rms value of a noise signal can also be obtained in the frequency domain using the following relationship:

$$V^2_{n(rms)} = \int_0^{\infty} V_n^2(f)\, df \tag{9.19}$$

More generally, the rms noise within a specified frequency range is obtained by integrating the noise spectral density over that frequency range. For example, the rms voltage noise over the frequency range $f_1 < f < f_2$ is

$$\int_{f_1}^{f_2} V_n^2(f)\, df \tag{9.21}$$

Finally, $V_n^2(f)$ is rigorously defined as the Fourier transform of the autocorrelation function of the time-domain signal, $v_n(t)$.

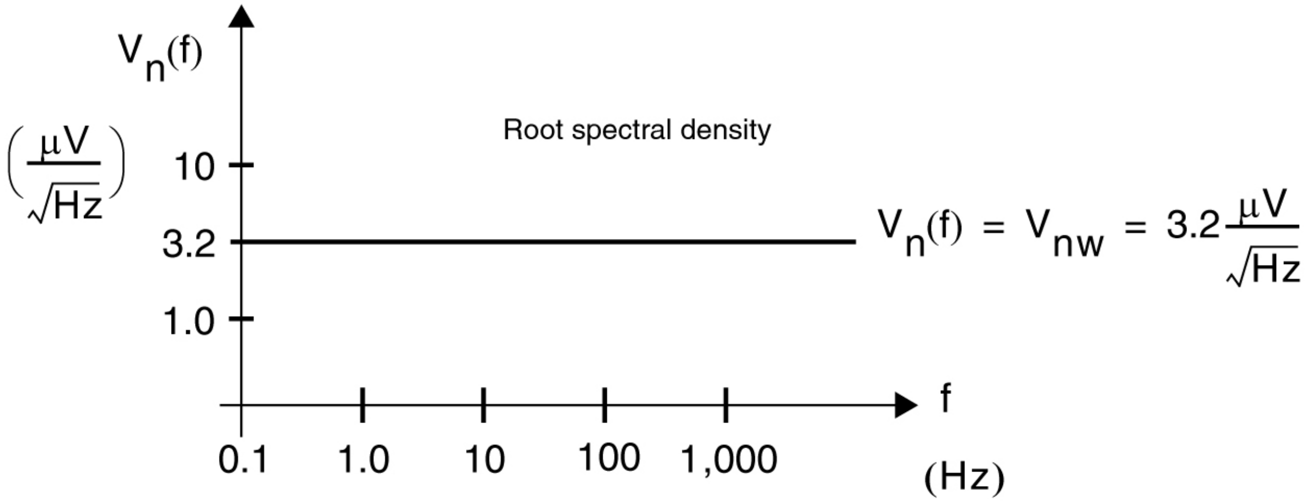
9.2.2 White noise

One common type of noise is *white noise*. A noise signal is said to be white if its spectral density is constant over a given frequency. In other words, a white noise signal would have a flat spectral density, as shown in Fig. 9.4, where $V_n(f)$ is given by

$$V_n(f) = V_{nw} \tag{9.22}$$

and V_{nw} is a constant value.

It appears that white noise has infinite power, but this is not happening in practice as a finite capacitance is always present to band-limit the noise.



Chapter 9 Figure 04

9.2.3 1/f or flicker noise

Another common noise shape is that of *1/f, or flicker,* noise.[3] The spectral density, $V_n^2(f)$, of 1/f noise is approximated by

$$V_n^2(f) = \frac{k_v^2}{f} \tag{9.23}$$

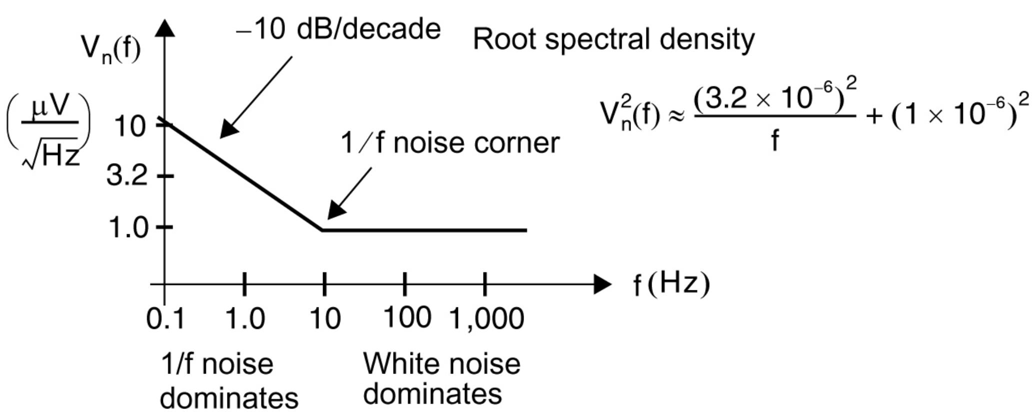
where k_v is a constant. Thus, the *spectral density is inversely proportional to frequency,* and hence the term "1/f noise." In terms of root spectral density, 1/f noise is given by

$$V_n(f) = \frac{k_v}{\sqrt{f}} \tag{9.24}$$

Substituting (9.23) into (9.21) gives the power of a 1/f noise source over a finite frequency range $f_1 < f < f_2$.

$$\int_{f_1}^{f_2} \frac{k_v^2}{f} df = k_v^2 \ln\left(\frac{f_2}{f_1}\right) \tag{9.25}$$

Hence, the noise power in every decade range of frequencies is equal to $k_v^2 \ln(10) \cong 2.3k_v^2$. Integration of 1/f noise all the way down to dc yields infinite power, but in practical cases, finite and reasonable values are obtained even when the lower limit of integration is very low.



Chapter 9 Figure 05

9.2.4 Filtered noise

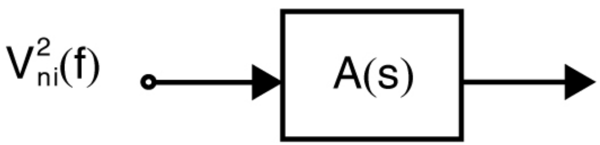
Consider the case of a noise signal, $V_{ni}(f)$, being filtered by the transfer function $A(s)$, as shown in Fig. 9.6. Here, $A(s)$ represents a linear transfer function as a result of some circuit amplification, filtering, or both. The following relationship between the input and output signals can be derived using the definition of the spectral density.

$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f) \tag{9.26}$$

$$V_{no}(f) = |A(j2\pi f)| V_{ni}(f)$$

Transfer function shapes the root spectral density

total output mean-squared value is given by $V_{no(rms)}^2 = \int_0^\infty |A(j2\pi f)|^2 V_{ni}^2(f)\,df$

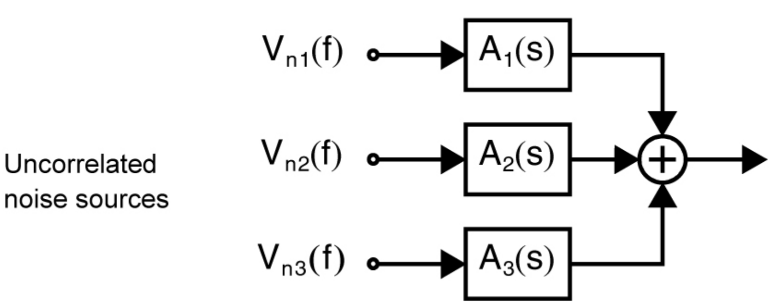


$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f)$$

$$V_{no}(f) = |A(j2\pi f)| V_{ni}(f)$$

Chapter 9 Figure 06

When multiple uncorrelated noise are filtered and summed together:



$$V_{no}(f) = \left(\sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f) \right)^{1/2}$$

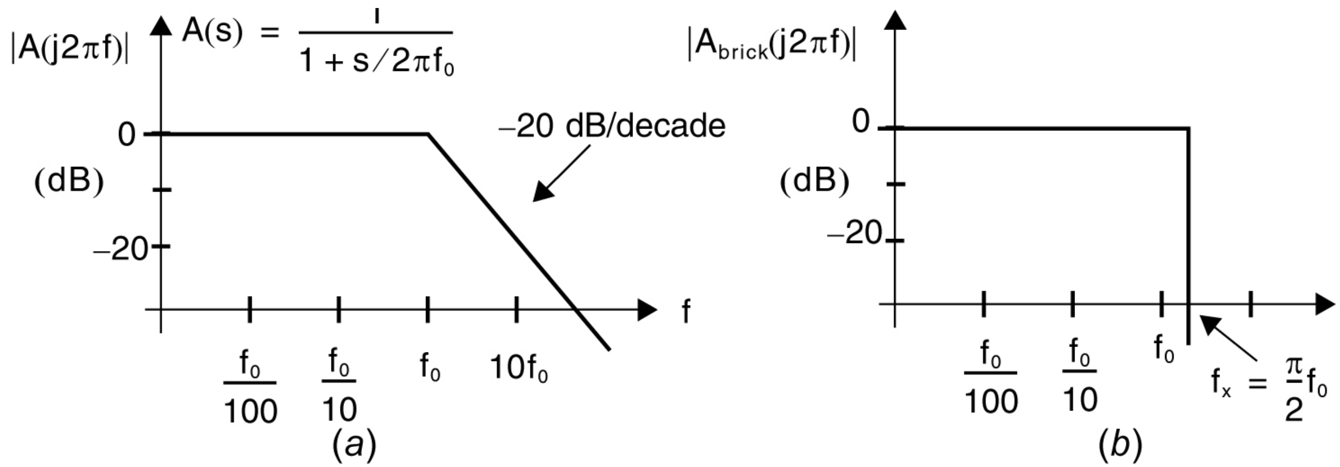
Chapter 9 Figure 07

9.2.5 Noise bandwidth

noise bandwidth of a given filter is equal to the frequency span of a brick-wall filter that has the same rms output noise as the given filter has when white noise is applied to both filters, assuming the same peak gain for both filters. In other words, given a filter response with peak gain A_0, the noise bandwidth is the width of a rectangular filter that has the same area and peak gain, A_0, as the original filter.

$$V^2_{no(rms)} = \int_0^{\infty} \frac{V^2_{nw}}{1+\left(\frac{f}{f_0}\right)^2} df = V^2_{nw} f_0 \arctan\left(\frac{f}{f_0}\right)\Bigg|_0^{\infty} = \frac{V^2_{nw}\pi f_0}{2}$$

$$f_x = \frac{\pi f_0}{2}$$



Chapter 9 Figure 09

The advantage of knowing the noise bandwidth of a filter is that, when white noise is applied to the filter input, the total output noise mean-squared value is easily calculated by multiplying the spectral density by the noise bandwidth. Specifically, in the first-order case just described, the total output noise mean-squared value, $V^2_{no(rms)}$, is equal to

$$V^2_{no(rms)} = V^2_{nw} f_x = V^2_{nw}\left(\frac{\pi}{2}\right) f_0 \tag{9.41}$$

9.3 Noise models for circuit elements

There are three main fundamental noise mechanisms: thermal, shot, and flicker.

Thermal noise is due to the thermal excitation of charge carriers in a conductor. This noise has a white spectral density and is proportional to absolute temperature. It is not dependent on bias conditions (dc bias current) and it occurs in all resistors (including semiconductors) above absolute zero temperature. Thus, thermal noise places fundamental limits on the dynamic range achievable in electronic circuits.

Shot noise This noise occurs because the dc bias current is not continuous and smooth but instead is a result of pulses of current caused by the flow of individual carriers. As such, *shot noise is dependent on the dc bias current*. It can also be modelled as a white noise source. Shot noise is also typically larger than thermal noise and is sometimes used to create white noise generators.

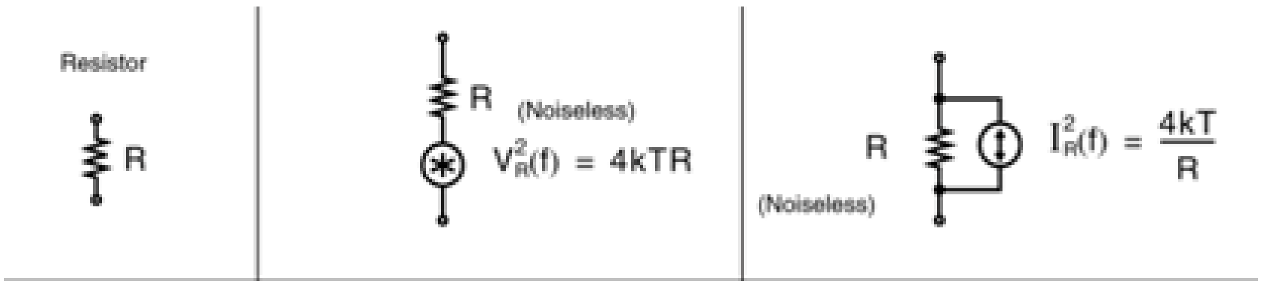
Flicker noise is the least understood of the three noise phenomena. It is found in all active devices as well as in carbon resistors,[4] but it occurs only when a dc current is flowing. Flicker noise usually arises due to traps in the semiconductor, where carriers that would normally constitute dc current flow are held for some time period and then released. Flicker noise is also commonly referred to as 1/f noise since it is well modelled as having a $1/f^{\alpha}$ spectral density, where α is between 0.8 and 1.3. Although both bipolar and MOSFET transistors have flicker noise, it is a significant noise source in MOS transistors, whereas it can often be ignored in bipolar transistors.

9.3.1 Resistors

The major source of noise in resistors is thermal noise. As just discussed, it appears as white noise and can be modelled as a voltage source, $V_R(f)$, in series with a noiseless resistor. With such an approach, the spectral density function, $V_R^2(f)$, is found to be given by

$$V_R^2(f) = 4kTR \tag{9.47}$$

where k is Boltzmann's constant (1.38×10^{-23} JK^{-1}), T is the temperature in Kelvins, and R is the resistance value.



An alternate model can be derived by finding the Norton equivalent circuit. Specifically, the series voltage noise source, $V_R(f)$, can be replaced with a parallel current noise source, $I_R(f)$, given by

$$I_R^2(f) = \frac{V_R^2(f)}{R^2} = \frac{4kT}{R} \tag{9.49}$$

9.3.2 Diodes

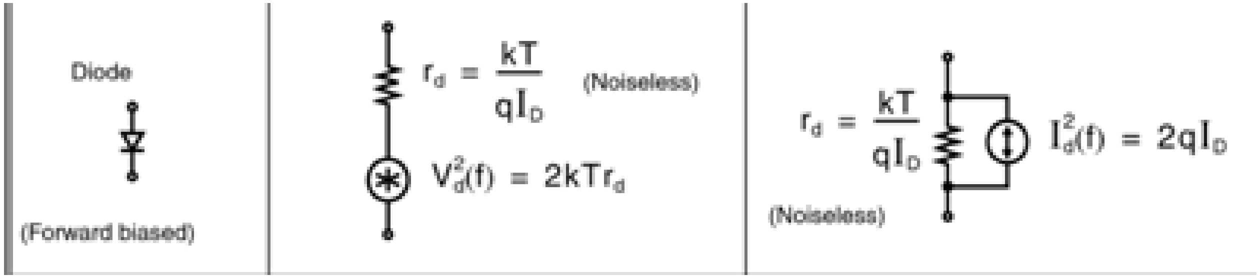
Shot noise is typically the dominant noise in diodes and can be modelled with a current source in parallel with the small-signal resistance of the diode, as Fig. 9.11 shows. The spectral density function of the current source is found to be given by

$$I_d^2(f) = 2qI_D \tag{9.50}$$

where q is one electronic charge $(1.6 \times 10^{-19}\ \mathrm{C})$ and I_D is the dc bias current flowing through the diode. The small-signal resistance of the diode, r_d, is given by the usual relationship,

$$r_d = \frac{kT}{qI_D} \tag{9.51}$$

The Thévenin equivalent circuit can also be used, as shown in Fig. 9.11. Note that the small-signal resistance, r_d, is used for small-signal modelling and is not a physical resistor; hence, r_d does not contribute any thermal noise.



9.3.3 Bipolar transistors

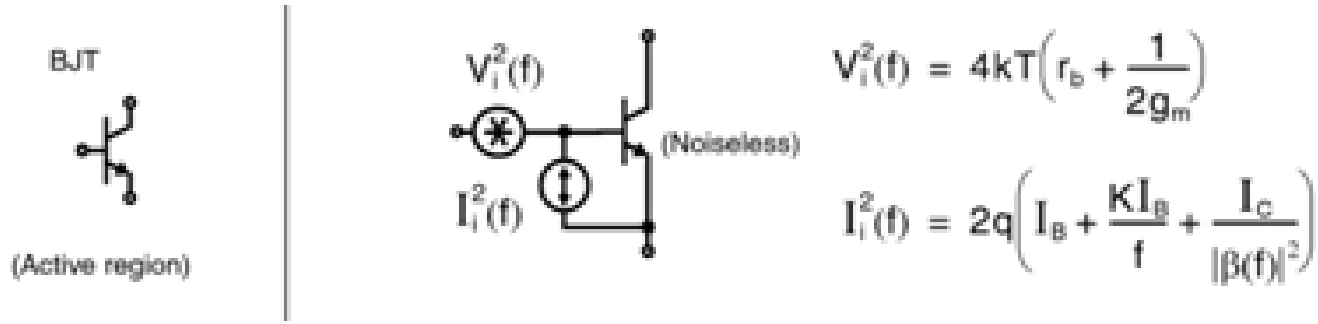
The noise in bipolar transistors is due to the shot noise of both the collector and base currents, the flicker noise of the base current, and the thermal noise of the base resistance. A common practice is to combine all these noise sources into two equivalent noise sources at the base of the transistor, as shown in Fig. 9.11. Here, the equivalent input voltage noise, $V_i(f)$, is given by

$$V_i^2(f) = 4kT\left(r_b + \frac{1}{2g_m}\right) \tag{9.52}$$

where the r_b term is due to the thermal noise of the base resistance and the g_m term is due to collector-current shot noise referred back to the input. The equivalent input current noise, $I_i(f)$, equals

$$I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{|\beta(f)|^2}\right) \tag{9.53}$$

where the $2qI_B$ term is a result of base-current shot noise, the KI_B/f term models 1/f noise (K is a constant dependent on device properties), and the I_C term is the input-referred collector-current shot noise (often ignored).



9.3.4 MOSFE transistors

The dominant noise sources for active MOSFET transistors are flicker and thermal noise, as shown in Fig. 9.11. The flicker noise is modelled as a voltage source in series with the gate of value

$$V_g^2(f) = \frac{K}{WLC_{ox}f} \tag{9.54}$$

where the constant K is dependent on device characteristics and can vary widely for different devices in the same process. The variables W, L, and C_{ox} represent the transistor's width, length, and gate capacitance per unit area, respectively. *The 1/f no ise is inversely pr oportional to the tran sistor ar ea,* WL, so larger devices have less 1/f noise. In MOSFET circuits, 1/f noise is extremely important because it typically dominates at low frequencies

1/f noise constant K is smaller for pMOS than nMOS since holes are less likely to be trapped than electrons. So, pMOS input differential pair is desired if 1/f noise to be reduced.

The derivation of the thermal noise term is straightforward and is due to the resistive channel of a MOS transistor in the active region.

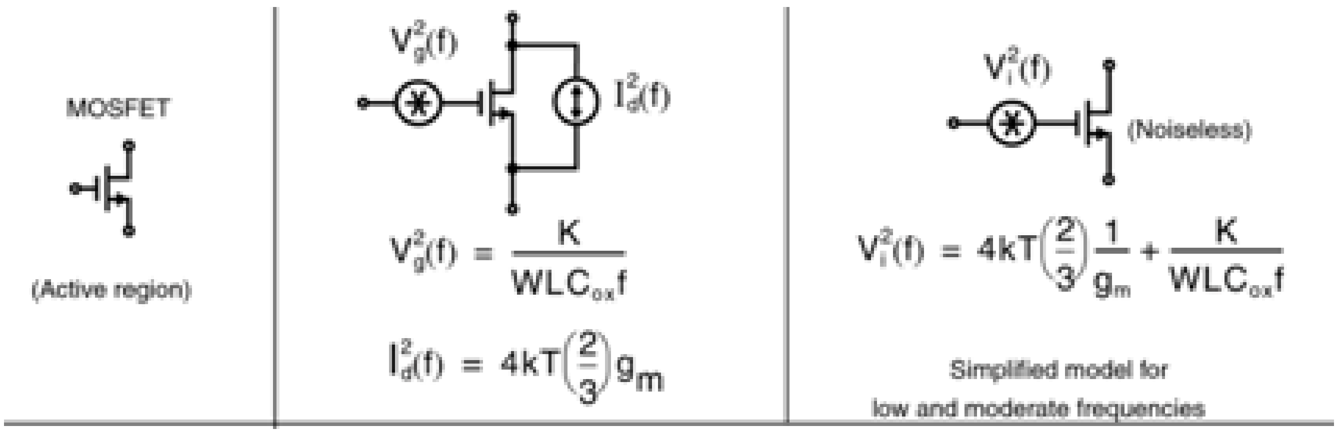
$$I_d^2(f) = 4kT\gamma g_m$$

For the case $V_{DS} = V_{GS} - V_T$ and assuming a long channel device, $\gamma = 2/3$. However, for short gate-length devices much higher values of γ may be observed. Note that the white noise parameter γ is different from the body-effect parameter γ.

9.3.4 MOSFE transistors

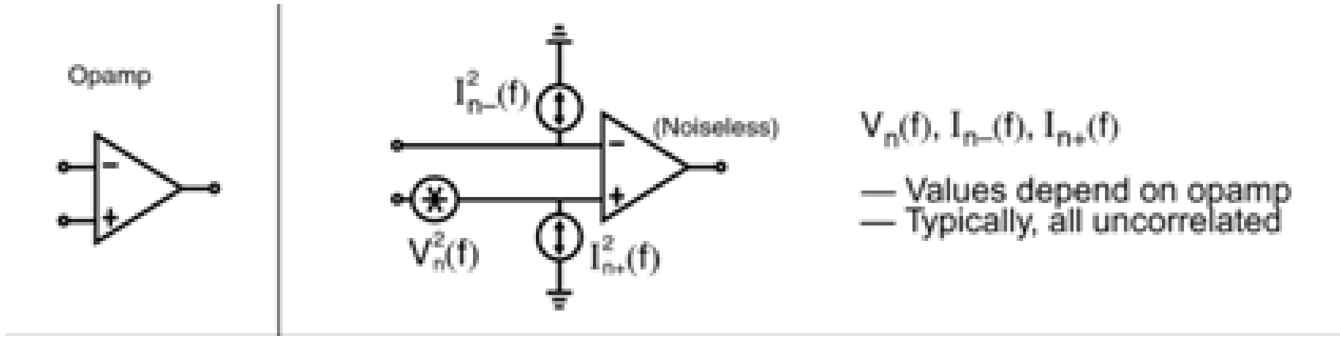
Noise analysis of MOS transistors may be simplified by transforming the current noise to an equivalent gate voltage noise, that is $I_d(f) = g_m V_i(f)$.

In this way, there is only one voltage noise source at the gate. However, note that this assumes the gate current equal to 0, which is valid at low and moderate frequencies. (At high frequencies, an noticeable amount of current may flow on C_{gs}).

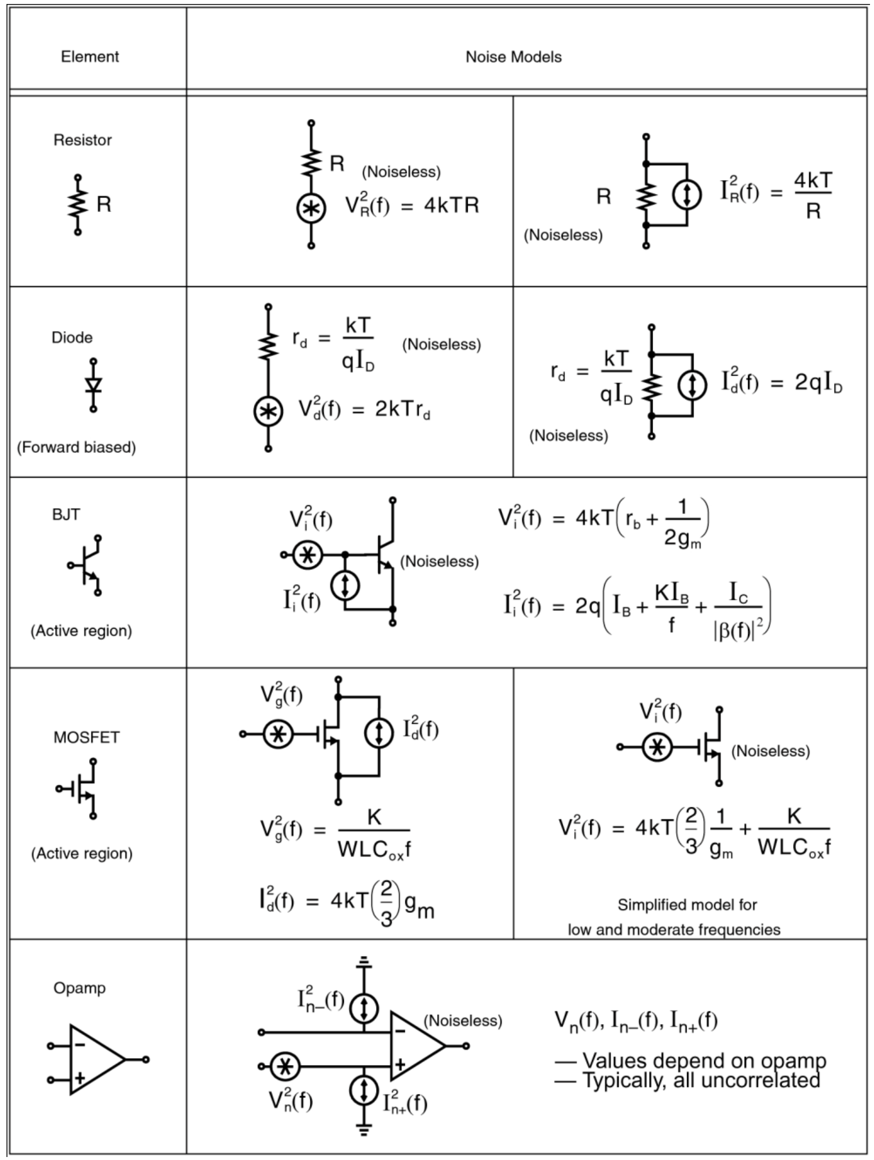


9.3.5 OpAmps

Noise in opamps is modelled using three uncorrelated input-referred noise sources, as shown in Fig. 9.11. With an opamp that has a MOSFET input stage, the current noises can often be ignored at low frequencies since their values are small. However, for bipolar input stages, all three noise sources are typically required,



9.3 Noise models for circuit elements



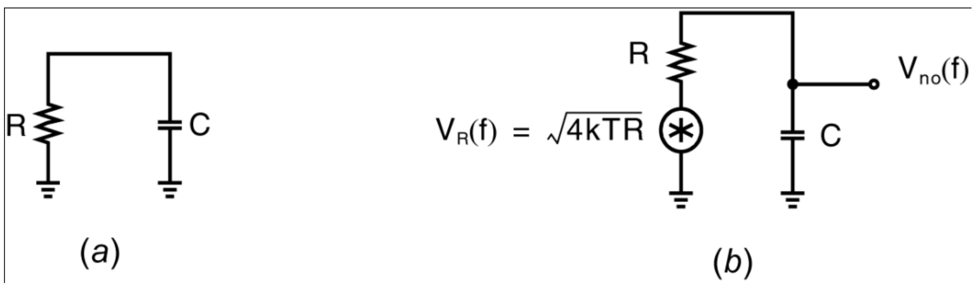
Chapter 9 Figure 11

9.3.6 Capacitors and inductors

Capacitors/inductors do not generate any noise, but they do accummulate noise generated by other noise sources.

Consider a capacitance, C, in parallel with a resistor of arbitrary size, R, as shown in Fig. 9.13(*a*). The equivalent circuit for noise analysis is shown in Fig. 9.13(*b*). To determine the total noise mean-squared value across the capacitor, we note that $V_{no}(f)$ is simply a first-order, low-pass, filtered signal with $V_R(f)$ as the input. Therefore, we recognize that the noise bandwidth is given by $(\pi/2)f_0$ as in (9.41), and since the input has a white spectral density, the total output mean-squared value is calculated as

$$
\begin{aligned}
V^2_{no(rms)} &= V^2_R(f)\left(\frac{\pi}{2}\right)f_0 = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right) \\
V^2_{no(rms)} &= \frac{kT}{C}
\end{aligned}
\qquad (9.59)
$$



Chapter 9 Figure 13

So the rms noise voltage across a capacitor is independent of the resistor R, but only C.

Finally, it should be mentioned that the equivalent noise current mean-squared value in an inductor of value L connected only to resistors is given by (see Problem 9.19)

$$
I^2_{no(rms)} = \frac{kT}{L} \qquad (9.62)
$$

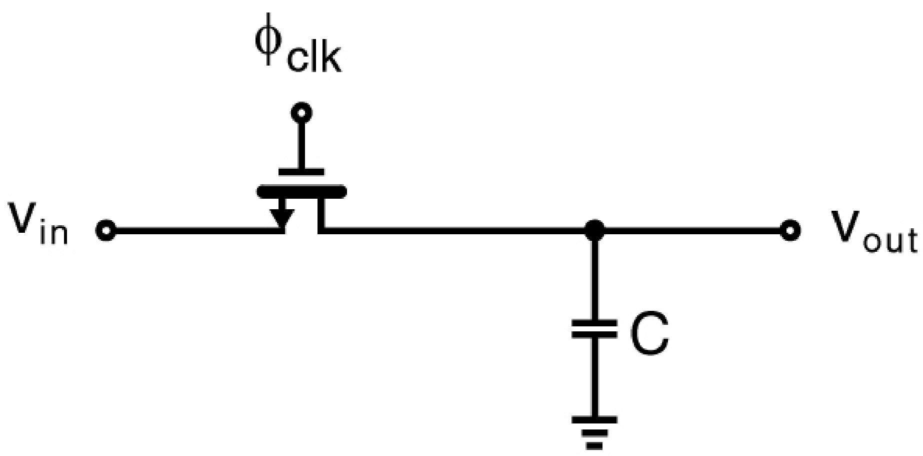
9.3.7 Sampled signal noise

For a sample and hold circuit, when the clock goes low, the transistor turns off and in ideal case the input voltage signal at that instance would be held on capacitance C.

However, when thermal noise is present, the resistance when the transistor is switched on causes voltage noise on the capacitor with an rms value of $\sqrt{kT/C}$.

So when the switch is turned off, the noise as well as the desired signal is held on the capacitor, so called the sample noise.

The sampled noise voltage does not depend on the sampling rate and is independent from sample to sample.



Chapter 9 Figure 14

9.3.8 Input referred noise

In general, the noise voltage at a particular node will be a superposition of multiple filtered un-correlated noise sources. In order to quantify the impact of all these noise sources on SNR, it is useful to know the total input-referred noise of the circuit.

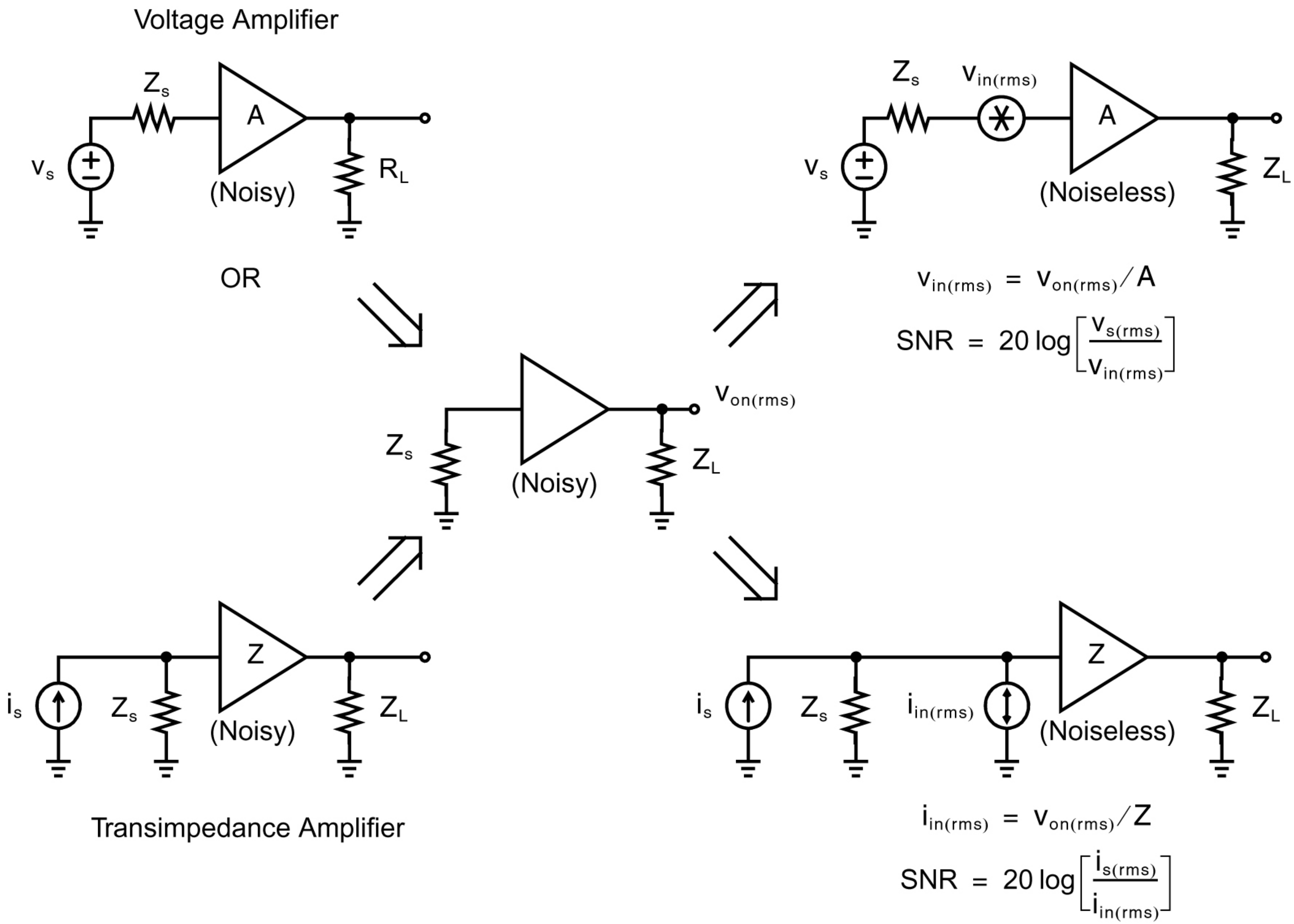
The input-referred noise of a circuit, if applied to the input of a noiseless copy of the circuit, results in the exact same output noise as when all of the circuit's noise sources were present.

It can be found by dividing the observed output noise by the Mid-band gain of the circuit using simulation or analysis as shown in the figure:

$v_{in(rms)} = v_{on(rms)}/A$ For voltage amplifier

$i_{in(rms)} = v_{on(rms)}/Z$ For trans-impedance amplifier

In the general, the output noise $V_{on(rms)}$ depends on the source and load impedance Z_s and Z_L as well, so they should be taken into account.



Chapter 9 Figure 15

Example 9.9 (page 385)

Two voltage amplifiers (each having very large input impedance and small output impedance) are available: one with a gain of 3 V/V and 3 μV_{rms} noise observed at the output; the other with a gain of 8 V/V and 6 μV_{rms} noise observed at its output. What is the input-referred noise of each amplifier? If the two amplifiers are to be placed in series to realize a gain of 24 V/V, in what order should they be placed in order to obtain the best noise performance? What is the resulting input-referred noise of the overall system?

$$V_{on(rms)}^2 = (1\ \mu V)^2(3 \cdot 8)^2 + (0.75\ \mu V)^2 8^2 = 0.61 \cdot 10^{-9}\ V^2$$

$$V_{on(rms)} = 24.7\ \mu V_{rms}$$

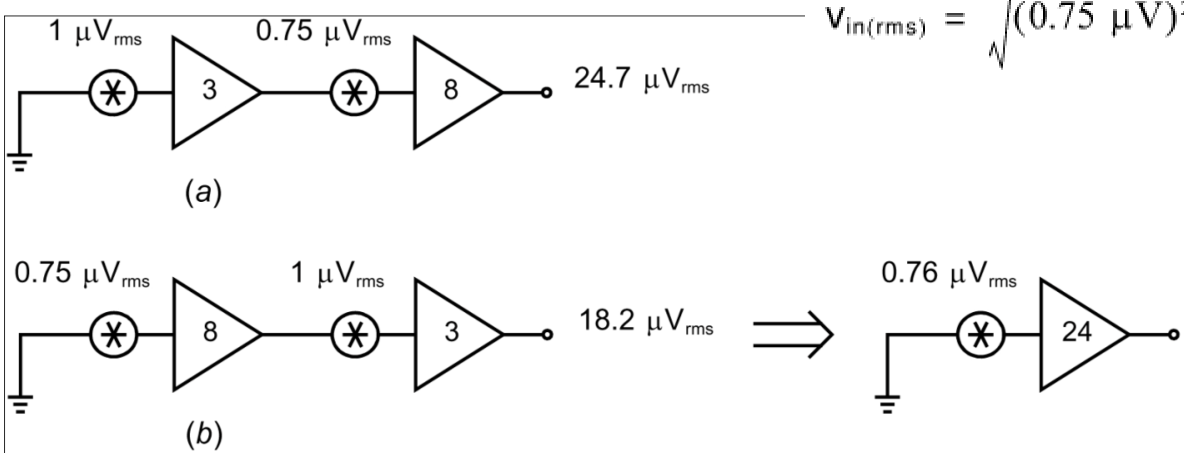
$$V_{on(rms)}^2 = (0.75\ \mu V)^2(8 \cdot 3)^2 + (1\ \mu V)^2 3^2 = 0.33 \cdot 10^{-9}\ V^2$$

$$V_{on(rms)} = 18.2\ \mu V_{rms}$$

Clearly, the second situation is preferable. The total input referred noise in this case is

$$V_{in(rms)} = 18.2\ \mu V_{rms}/(3 \cdot 8) = 0.76\ \mu V_{rms}$$

$$V_{in(rms)} = \sqrt{(0.75\ \mu V)^2 + \frac{(1\ \mu V)^2}{8^2}} = 0.76\ \mu V_{rms}$$



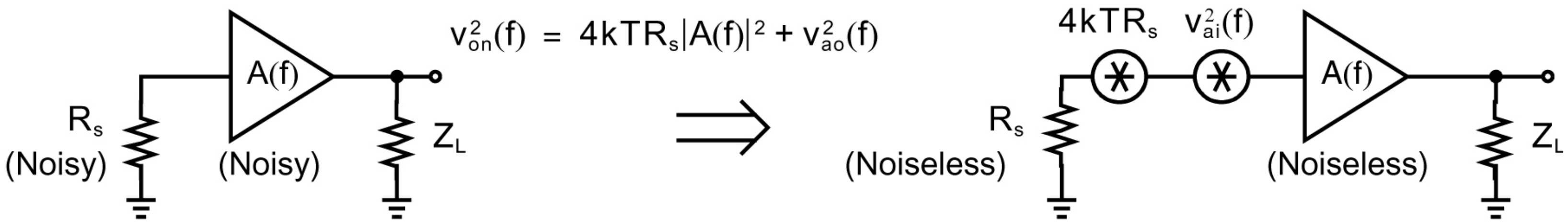
Chapter 9 Figure 16

So, larger gain in the first stage renders the later stage noise negligible.

9.3.8 Input referred noise: general case

The noise spectral density at the output of the circuit is generally the sum of noise contributed by the circuit itself, $v_{ao}(f)$, as well as thermal noise from the real part of the source impedance filtered by the circuit, $4kTR_s|A(f)|^2$. These quantities may be input referred as illustrated in Fig. 9.17 for a voltage amplifier where

$$v_{ai}^2(f) = \frac{v_{ao}^2(f)}{|A(f)|^2} \tag{9.65}$$



Chapter 9 Figure 17

9.4 Noise analysis example

Current noise sources are used for R1 and Rf, whereas a voltage noise source is used for R2. This way simplifies the analysis.

Assuming all noise sources are un-correlated, find the output voltage noise, $V^2_{no1}(f)$, due only to I_{n1}, I_{nf}, and I_{n-}. These current sources add together and go through the parallel of Rf and Cf.

$$V^2_{no1}(f) = [I^2_{n1}(f) + I^2_{nf}(f) + I^2_{n-}(f)]\left|\frac{R_f}{1 + j2\pi f R_f C_f}\right|^2$$

Using superposition principle, the output oise due to three noise sources at the positive OpAmp terminal (converting In+ to a voltage source by multiplying R2).

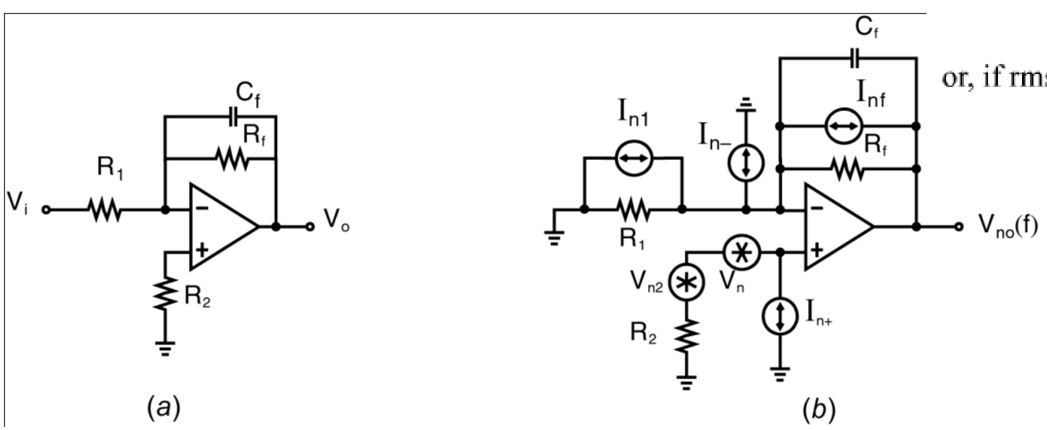
$$V^2_{no2}(f) = [I^2_{n+}(f)R^2_2 + V^2_{n2}(f) + V^2_n(f)]\left|1 + \frac{R_f/R_1}{1 + j2\pi f C_f R_f}\right|^2$$

Finally, the total output noise mean-squared value is simply the sum

$$V^2_{no}(f) = V^2_{no1}(f) + V^2_{no2}(f)$$

or, if rms values are found,

$$V^2_{no(rms)} = V^2_{no1(rms)} + V^2_{no2(rms)}$$



Chapter 9 Figure 18

If output voltage 1V and rms noise of 77uV

$$SNR = 20\log\left(\frac{1\ V}{77\ \mu V}\right) = 82\ dB$$

9.4 Noise analysis example (MOS diff. pair)

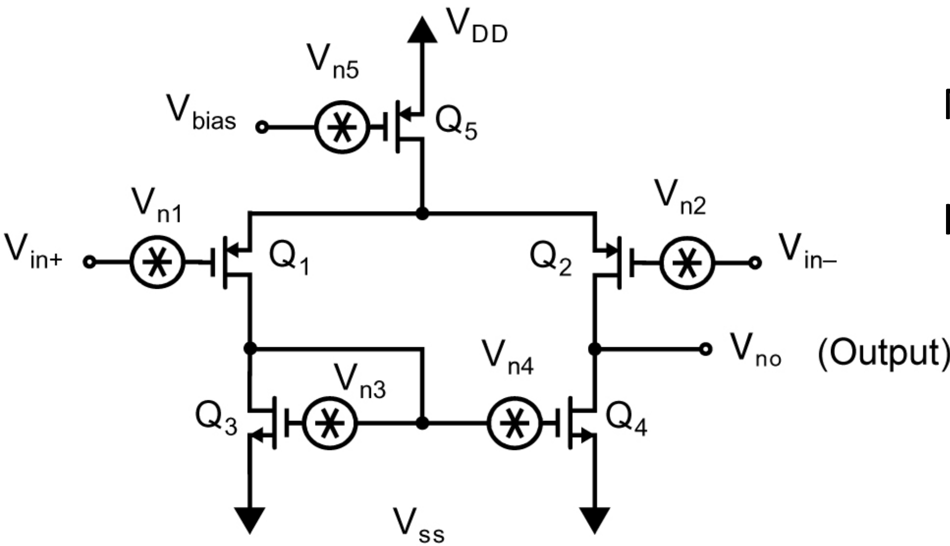
As seen before, if the input stage has a good gain, then its noise will dominate the overall noise of a two-stage OpAmp.

Each transistor has been modeled using an equivalent voltage noise source. We need to find the gain from each noise source to the output node.

$$\left|\frac{V_{no}}{V_{n1}}\right| = \left|\frac{V_{no}}{V_{n2}}\right| = g_{m1}R_o \quad \text{where } R_o \text{ is the output impedance seen at } V_{no}.$$

Vn3 and Vn4 generates noise current that goes to the output, so $\left|\frac{V_{no}}{V_{n3}}\right| = \left|\frac{V_{no}}{V_{n4}}\right| = g_{m3}R_o$

Finally, the noise gain from Vn5 to output can be found by $\left|\frac{V_{no}}{V_{n5}}\right| = \frac{g_{m5}}{2g_{m3}}$



Chapter 9 Figure 20

Neglecting Vn5 (gain is small), output noise is

$$V_{no}^2(f) = 2(g_{m1}R_o)^2V_{n1}^2(f) + 2(g_{m3}R_o)^2V_{n3}^2(f)$$

Input-referred noise is

$$V_{neq}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2(f)\left(\frac{g_{m3}}{g_{m1}}\right)^2$$

9.4 Noise analysis example (MOS diff. pair)

Thus, for the white noise portion of $V_{n1}(f)$ and $V_{n3}(f)$, we make the substitution $V_{ni}^2(f) = 4kT\gamma\left(\frac{1}{g_{mi}}\right)$

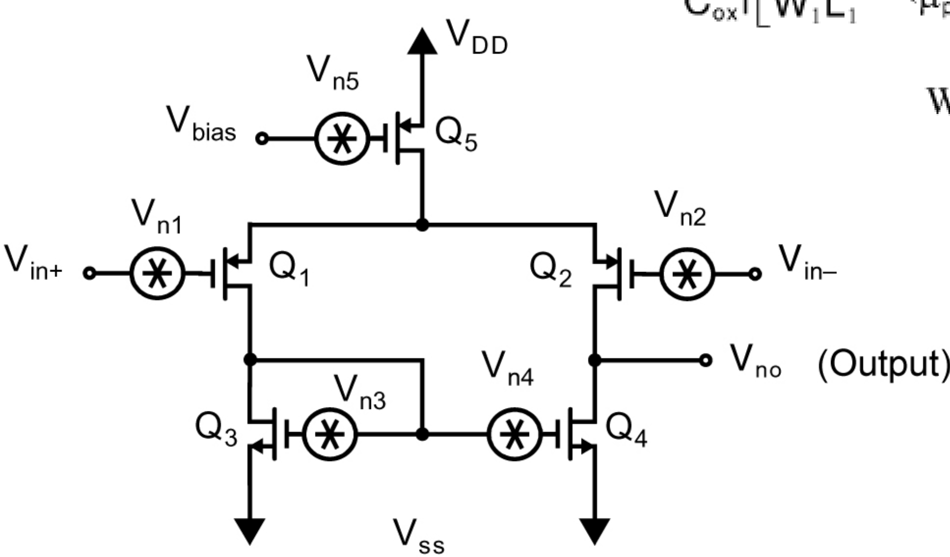
resulting in $V_{neq}^2(f) = 2 \cdot 4kT\gamma\left(\frac{1}{g_{m1}}\right) + 2 \cdot 4kT\gamma\left(\frac{g_{m3}}{g_{m1}}\right)^2\left(\frac{1}{g_{m3}}\right)$

So, g_{m1} should be made large to minimize thermal noise and g_{m3} small. For fixed DC current I_{D5}, this means small V_{eff1} and large V_{eff3}.

Next, we consider the effects of 1/f, or flicker, noise, $V_{ni}^2(f) = \frac{K_i}{W_i L_i C_{ox} f}$

resulting in $V_{ni}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2(f)\left[\frac{(W/L)_3 \mu_n}{(W/L)_1 \mu_p}\right]$ $\leftarrow$ $g_{mi} = \sqrt{2\mu_i C_{ox}\left(\frac{W}{L}\right)_i I_{Di}}$

$$V_{ni}^2(f) = \frac{2}{C_{ox} f}\left[\frac{K_1}{W_1 L_1} + \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{K_3 L_1}{W_1 L_3^2}\right)\right] \qquad (9.109)$$



Chapter 9 Figure 20

We note some points for 1/f noise here:

1. When L1=L3, , nMOS loads dominate the noise as $\mu_n > \mu_p$ and $K_3 > K_1$,
2. Taking L3 longer helps.
3. Noise independent of W3 (but thermal noise is increased).
4. Taking W1 wider helps both 1/f and thermal noise.
5. Taking L1 longer increase the noise of from the second term which may be dominant.

9.5 Dynamic range performance

Whereas noise limits the value of the smallest useful signals, linearity limits the value of the largest useful signals that can be processed by a circuit.

Thus, noise and linearity together determine the useful dynamic range of the circuit.

Harmonic distortion and total harmonic distortion are useful measurements of the linearity of a circuit.

9.5.1 Total harmonic distortion

If a sinusoidal signal is applied to a nonlinear system, the output signal will have frequency components at harmonics of the same input.

Specifically, consider a nonlinear system[9] with an input signal, $v_{in}(t)$, and an output signal, $v_o(t)$. The output signal can be written as a Taylor series expansion of the input signal:

$$v_o(t) = a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t) + a_4 v_{in}^4(t) + \cdots \tag{9.123}$$

Here, the linear term is a_1, whereas a_2, a_3, and a_4 characterize the second-, third-, and fourth-order distortion terms, respectively. In fully differential circuits, all even terms (i.e., a_2 and a_4) are small, so typically a_3 dominates and we approximate $v_o(t)$ as

$$v_o(t) \cong a_1 v_{in}(t) + a_3 v_{in}^3(t) \tag{9.124}$$

If $v_{in}(t)$ is a sinusoidal signal given by

$$v_{in}(t) = A\cos(\omega t) \tag{9.125}$$

the output signal can be shown to be approximated by

$$v_o(t) \cong a_1 A\cos(\omega t) + \frac{a_3}{4}A^3[3\cos(\omega t) + \cos(3\omega t)] \tag{9.126}$$

Since, typically, $(3/4)a_3 A^3 << a_1 A$, one usually approximates the linear component of the output signal as

$$H_{D1} = a_1 A \tag{9.128}$$

and the third-harmonic term as

$$H_{D3} = \frac{a_3}{4}A^3 \tag{9.129}$$

9.5.1 Total harmonic distortion

H_{D3}/H_{D1} is defined as the third-order harmonic distortion ratio, given by

$$HD_3 \equiv \frac{H_{D3}}{H_{D1}} = \left(\frac{a_3}{a_1}\right)\left(\frac{A^2}{4}\right) \tag{9.130}$$

The *total harmonic distortion* (THD) of a signal is defined to be the ratio of the total power of all second and higher harmonic components to the power of the fundamental for that signal. In units of dB, THD is found using

$$THD = 10\log\left(\frac{H_{D2}^2 + H_{D3}^2 + H_{D4}^2 + \cdots}{H_{D1}^2}\right) \tag{9.131}$$

Sometimes THD is presented as a percentage value. In this case,

$$THD = \frac{\sqrt{H_{D2}^2 + H_{D3}^2 + H_{D4}^2 + \cdots}}{H_{D1}} \times 100\,\% \tag{9.132}$$

Note that THD is always a function of the input signal amplitude, so a THD with the corresponding input amplitude needs to be reported. Second, in most cases only up to 5th harmonics needs to be considered.

The THD of a circuit deteriorates as the input signal amplitude is increased, as clearly shown from equation 9.130.

9.5.1 Total harmonic distortion

One difficulty with THD in reporting circuit performance is that often harmonic components falls outside the circuit usable bandwidth, thus the THD value is falsely improved.

For example, the harmonics of a 20MHz signal is already outside the passband of a 21MHz lowpass filter. In this case, the THD value will indicate much better linearity than would occur for a practical application.

Therefore, a THD measurement is straight-forward to perform, but does not work well in the important test of high-frequency signals near the upper passband limit of the circuit (at upper passband limit, the circuit linearity is usually worst).

One way to solve this is to use inter-modulation test.

9.5.2 Third order intercept point (IIP3)

We can use an intermodulation test to move the distortion term back near the frequency of the input signals.

Consider an intermodulation test, where the input signal consists of two equally sized sinusoidal signals and is written as

$$v_{in}(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t) \tag{9.135}$$

Presuming the input–output relationship of (9.123), the output signal can be shown to be approximated by

$$\begin{aligned} v_o(t) \cong & \left(a_1 A + \frac{9a_3}{4}A^3\right)[\cos(\omega_1 t) + \cos(\omega_2 t)] \\ & + \frac{a_3}{4}A^3[\cos(3\omega_1 t) + \cos(3\omega_2 t)] \\ & + \frac{3a_3}{4}A^3[\cos(2\omega_1 t + \omega_2 t) + \cos(2\omega_2 t + \omega_1 t)] \\ & + \frac{3a_3}{4}A^3[\cos(\omega_1 t - \Delta\omega t) + \cos(\omega_2 t + \Delta\omega t)] \end{aligned} \tag{9.136}$$

where $\Delta\omega$ is defined to be the difference between the input frequencies (i.e., $\Delta\omega \equiv \omega_2 - \omega$) which we assume to be small.

fourth line describes the distortion levels at two new frequencies that are close to the input frequencies (slightly below ω_1 and slightly above ω_2). As a result, for a narrowband or low-pass circuit, these two new distortion components (due to third-order distortion) fall in the passband and can be used to predict the third-order distortion term.

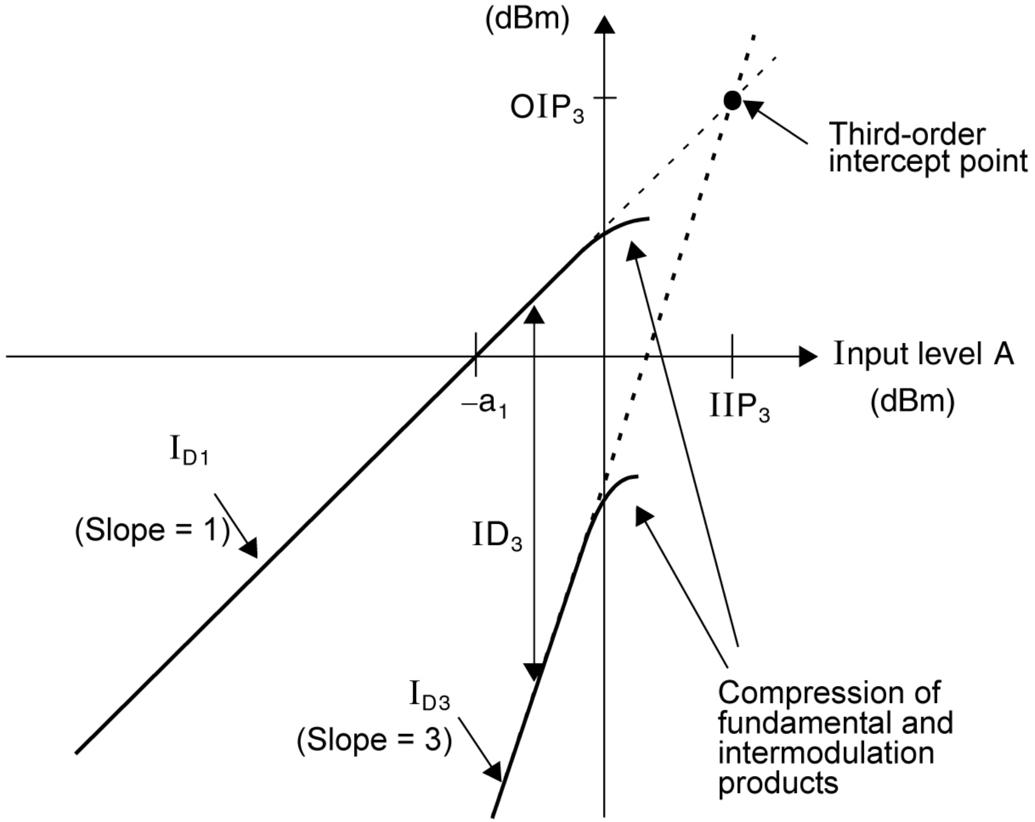
9.5.2 Third order intercept point (IIP3)

$$I_{D1} = a_1 A$$

$$I_{D3} = \frac{3a_3}{4} A^3$$

The ratio of these two is the third-order intermodulation value, given by

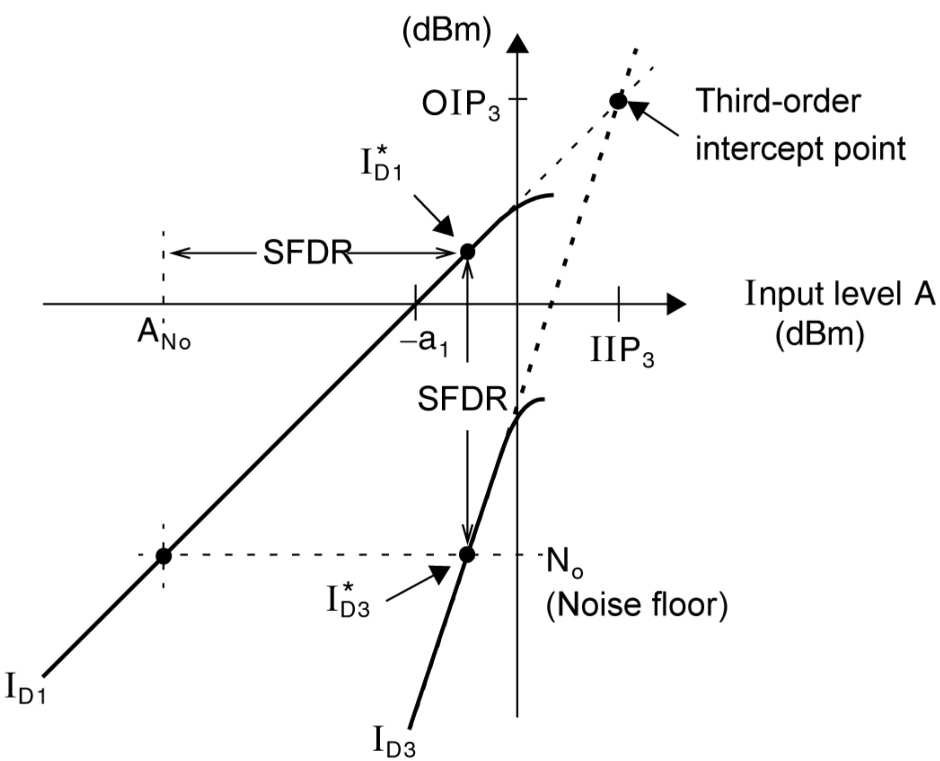
$$ID_3 = \frac{I_{D3}}{I_{D1}} = \left(\frac{a_3}{a_1}\right)\left(\frac{3A^2}{4}\right)$$



Chapter 9 Figure 24

9.5.3 Spurious free dynamic range

Spurious-free dynamic range (SFDR) is defined to be the signal-to-noise ratio when the power of the distortion equals the noise power. In an intermodulation test, the third order intermodulation products are the dominant distortion. In Fig. 9.25, the circuit's total output noise power is shown along the vertical axis as N_o. If a low enough signal level is used, I_{D3} will be well below the noise floor. However, since I_{D3} rises 3 dB for every 1 dB of signal-level increase, there will soon be a point where I_{D3} is equal to the noise power. As the figure shows, SFDR is defined to be the output SNR ratio when I_{D3} is equal to N_o. Alternatively, one can measure SFDR using the input-signal levels as the difference between the level that results in $I_{D3} = N_o$ and the level A_{No} that results in a fundamental output level equal to N_o.

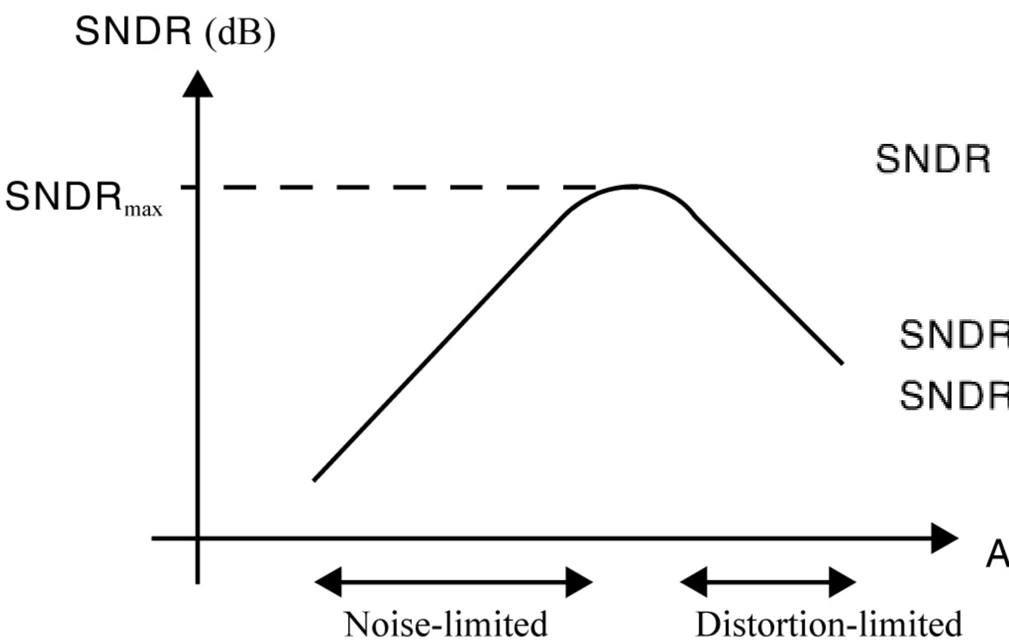


Chapter 9 Figure 25

9.5.4 Signal-to-Noise Ratio

The SNDR is defined as the ratio of the signal power to the total power in all noise and distortion components.

Unlike SFDR, SNDR is a function of the signal amplitude. For small amplitude, noise dominates over harmonics power, therefore we see an increasing SNDR typically. For large amplitude, harmonics power kick in and dominates over noise power. So a maximum SNDR is achieved at a certain input amplitude. As harmonics power increases faster than si9gnal power, SNDR will decrease eventually with larger inputs.



$$SNDR = 10 \log\left(\frac{V_f^2}{N_o + V_{h2}^2 + V_{h3}^2 + V_{h4}^2 + \cdots}\right)$$

$SNDR \cong 10 \log(V_f^2/N_o)$. For small signals

$SNDR \cong 10 \log(V_f^2/V_{h2}^2 + V_{h3}^2 + V_{h4}^2 + \cdots)$

For large signals

Chapter 9 Figure 26