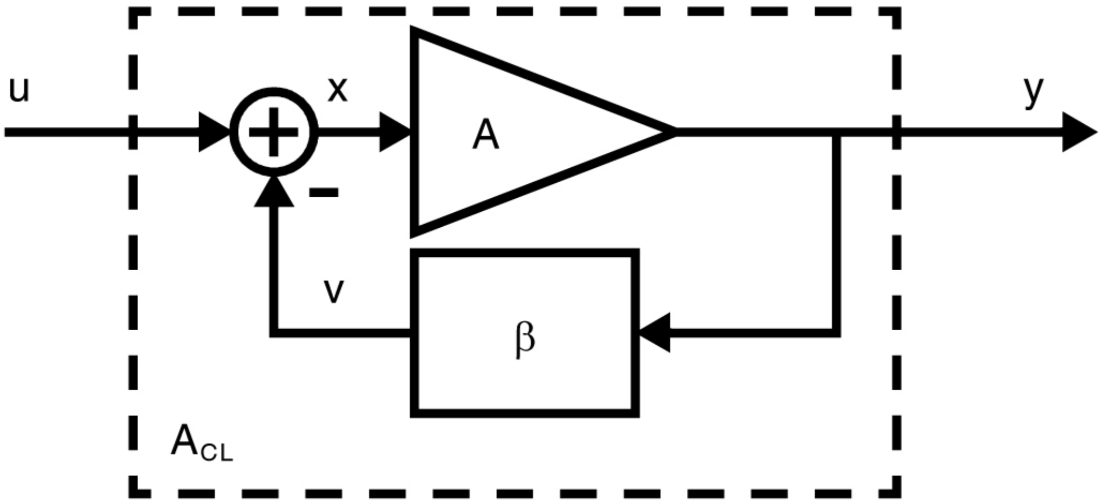
# Today's topic: feedback in amplifiers

Chapter 5

Feedback in amplifiers allows to construct almost ideal amplifiers from the poorly controlled and very non-ideal transistors.

# 5.1 Model of negative feedback




Chapter 5 Figure 01

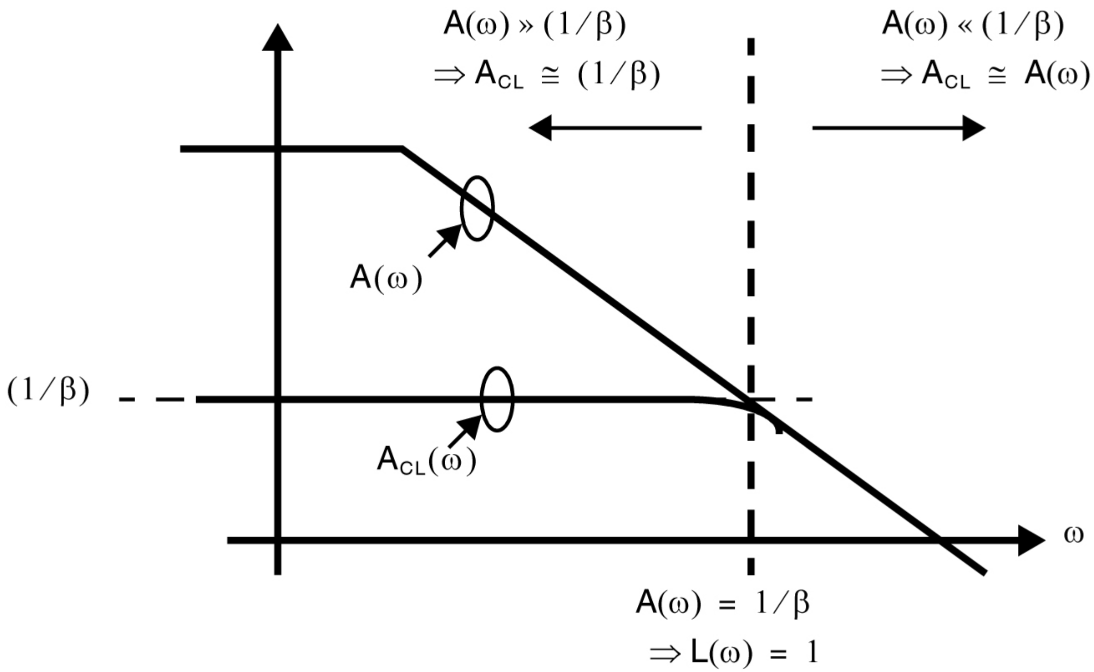
"loop gain,"

$$L \equiv A\beta$$

The "closed-loop" gain $$A_{CL} \equiv \frac{y}{u} = \frac{A}{1 + A\beta} = \frac{A}{1 + L}$$

$$A_{CL} \cong \frac{A}{L} = \frac{1}{\beta}$$ as long as $L \gg 1$.

# Bandwidth and linearity




Chapter 5 Figure 02

When considering 3dB bandwidth, that of the $A_{CL}(f)$ is usually much larger than that of the open-loop gain A(f). Note that it requires that feedback coefficient remains constant over the entire frequency range of consideration.

The previous model is linear, hence requires that the amplifiers in the linear range of operation. Note $x \cong \frac{u}{L}$ , so in feedback amplifier A is exposed to signal that are L times smaller than would the case without feedback, which makes it linear.

# 5.2.1 Stability criteria

Stability can be tested by checking that all poles of the closed loop system,

$$A_{CL}(s) = \frac{A(s)}{1 + A(s)\beta}$$

are in the left-half plane. However, this method is not often used because obtaining $A(s)$ with accuracy is diffi·cult.

Fortunately, stability can be check by using the loop gain L(s)= $A(s)\beta$, which can be obtained readily from circuit simulation.

Assuming the feedback amplifier has a constant and well-controlled gain, $\beta \le 1$.

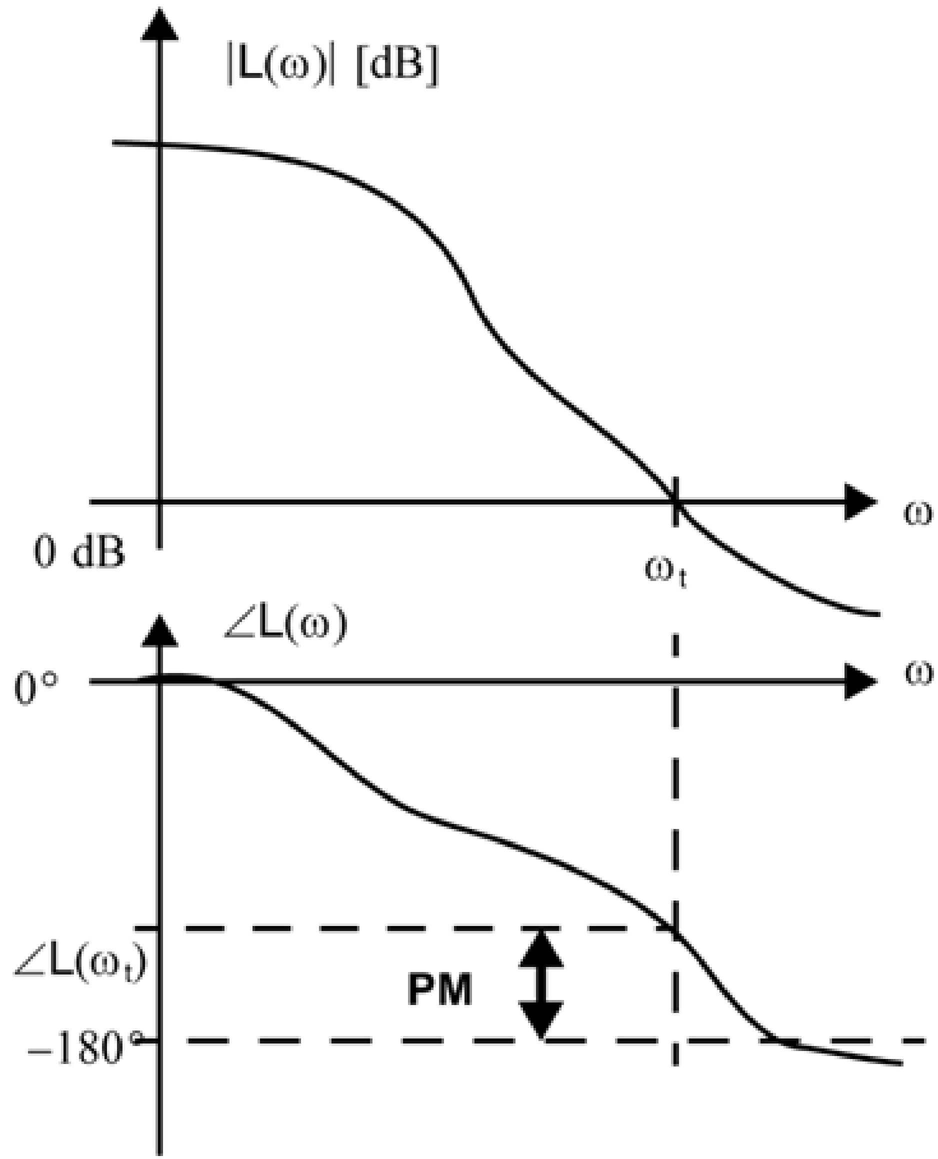
Magnitude of L(s) $$|L(\omega)| = |A(\omega)|\beta$$

Phase of L(s) $$\angle L(\omega) = \angle A(\omega) + \angle\beta = \angle A(\omega) + 0^\circ = \angle A(\omega)$$

A typical plot is shown in the next slide: please note the

$$|L(\omega_t)| = 1 \quad \text{“unity-gain frequency”}, \omega_t.$$

If $\angle L(\omega_t) > -180^\circ$ , then the feedback amplifier is stable.



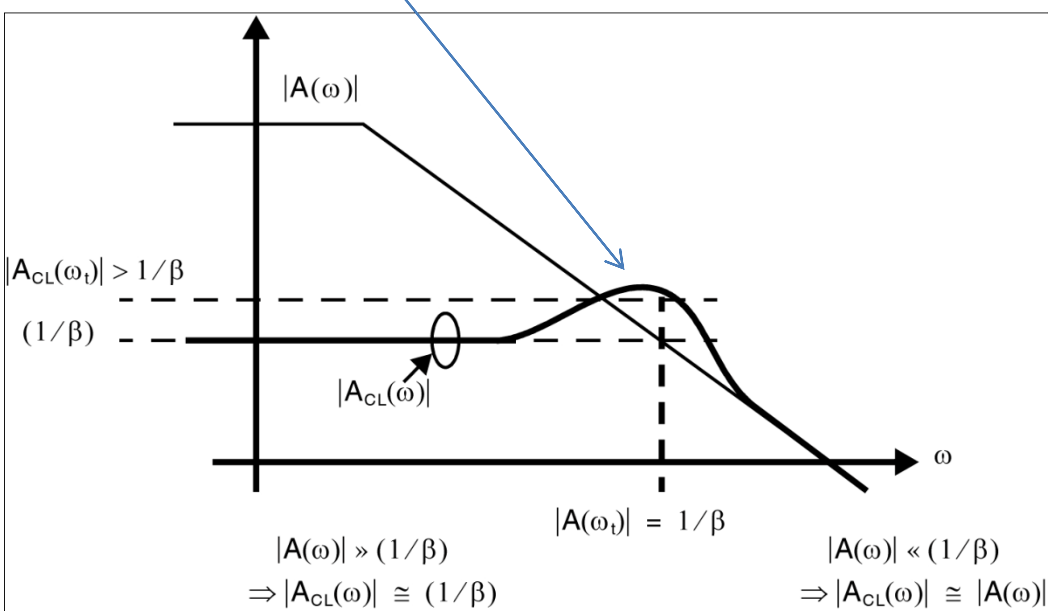
|L(ω)| [dB]
ω
0 dB
ω_t
∠L(ω)
0°
ω
∠L(ω_t)
PM
−180°

# 5.2.2 Phase margin

Phase margin provides a quantitative measure of how close a feedback system is to instability. It also gives the designer important information on the frequency and transient response of the feedback amplifier.

Definition of PM: $PM \equiv \angle L(\omega_t) + 180°$

We generally require a large phase margin between 45 and 90 degrees so that the feedback amplifier will not exhibit undesirable dynamic behavior. In fact, >60 degrees is preferred to avoid large frequency peaking and transient ringing. (see example 5.5 page 212)




Chapter 5 Figure 06

# 5.3 First and second order feedback system

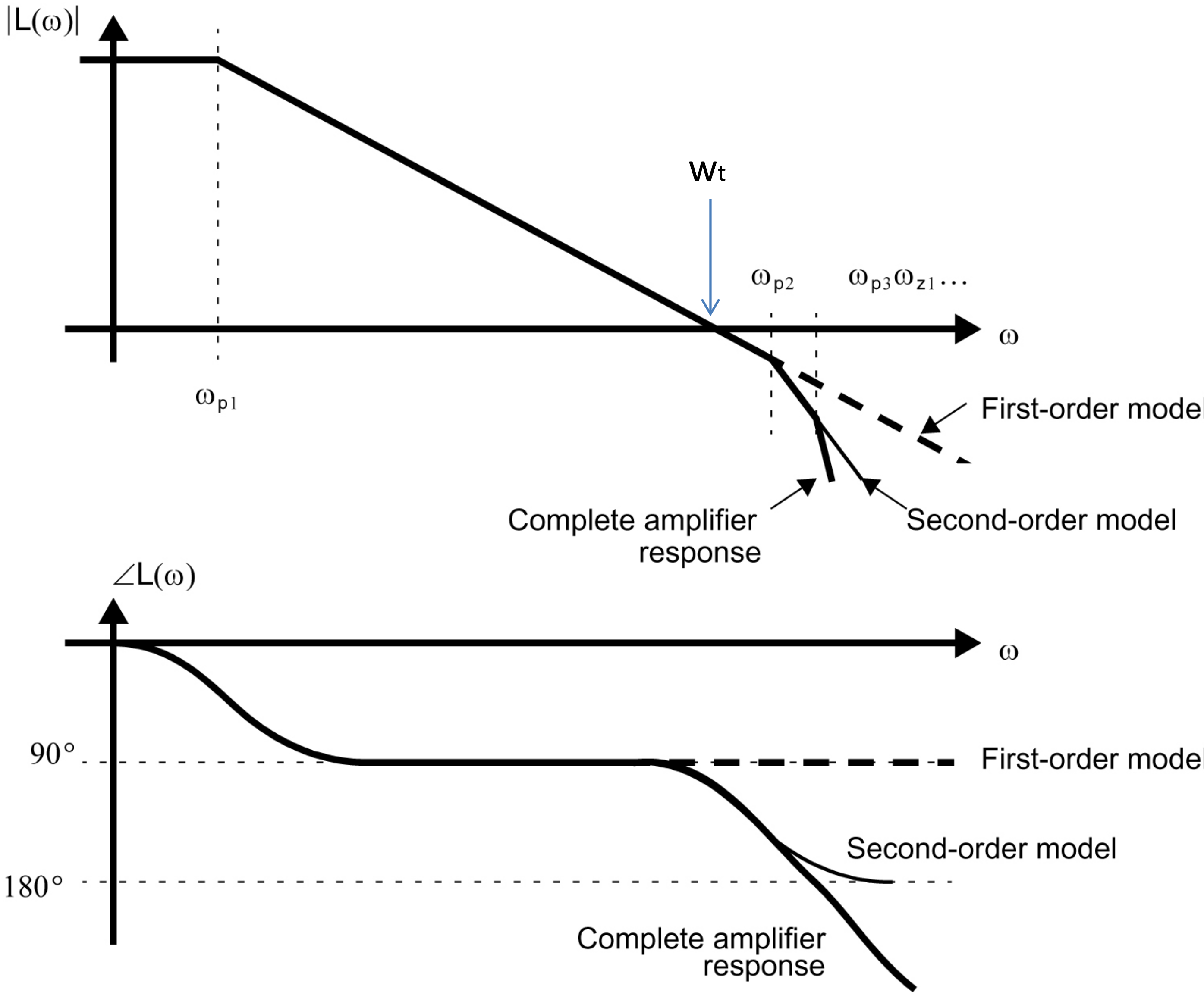
A practical amplifier has many poles and zeros, but it is often sufficient to model it with a simple first or second order transfer function.

Also, it is most important to accurately understand the amplifier's response for frequencies $w <= w_t$, which determines the amplifier's stability and in-band performance. Any zeros/poles at frequencies $w >> w_t$ may be neglected.

In the plot of next slide, it can be seen that a first-order model is sufficient to model the behavior of feedback amplifiers when $w_t << w_{p2}$, $w_{p3}$, $w_{z1}$,…

A second-order model is preferred when $w_t$ is close to $w_{p2}$, $w_{p3}$, $w_{z1}$, …

For cases where $w_t$ is even larger than $w_{p2}$, $w_{p3}$, $w_{z1}$, …, even the second order model is not accurate. But fortunately, these cases will have little phase margin and therefore are NOT of interest in practical designs.



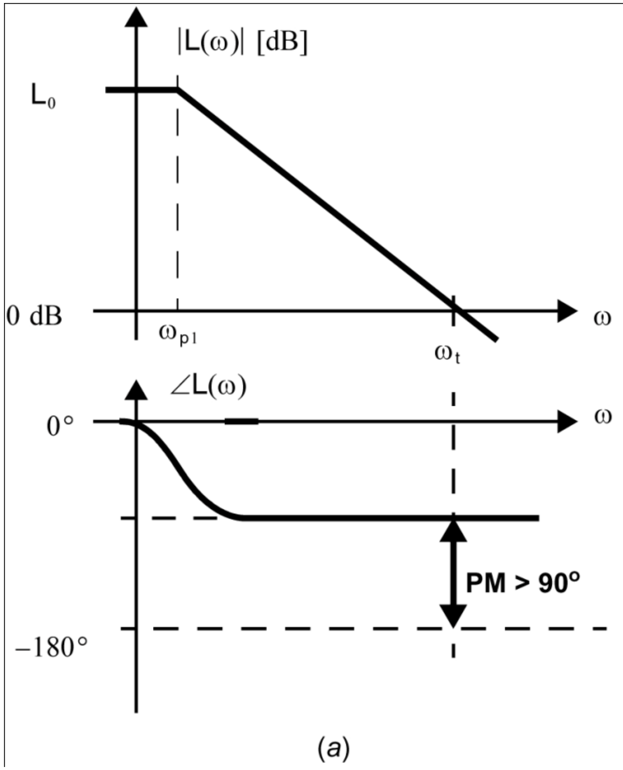

Chapter 5 Figure 07

# 5.3.1 First order feedback systems

A simple first-order model for the transfer function of a dominant-pole feedback amplifier, $A(s)\beta$ with dc gain $L_0$ and dominant pole frequency $\omega_{p1}$ is given by

$$L(s) = \frac{L_0}{(1 + s/\omega_{p1})} \qquad (5.22)$$

It is clear that first order feedback systems are absolutely stable with 90 degrees of PM for any $L_0$ or $w_{p1}$.




Chapter 5 Figure 08

assuming $\omega_t >> \omega_{p1}$: $\quad |L(\omega_t)| = 1 \cong \dfrac{L_0}{\omega_t/\omega_{p1}}$

the unity-gain frequency $\omega_t$, is the frequency at which $|L(\omega_t)| = 1$.

$$\omega_t \cong L_0\omega_{p1}$$

for the case in which $\omega \gg \omega_{p1}$, $\quad L(s) \cong \dfrac{\omega_t}{s}$

transfer function of the closed-loop amplifier

$$A_{CL}(s) \cong \frac{1}{\beta}\frac{1}{(1 + s/\omega_t)}$$

The –3dB frequency of the closed-loop amplifier is given by

$$\omega_{-3\ dB} \cong \omega_t$$

# Settling time

The settling time performance of integrated amplifiers is often an important design parameter, which defines how fast the output settles to the steady-state and is related to the step response.

Settling time is defined to be the time it takes for an amplifier to reach a specified percentage of the steady-state value when a step input is applied.

the step response of any first-order circuit is given by

$$v_{out}(t) = V_{step}(1 - e^{-t/\tau}) \qquad \tau = \frac{1}{\omega_{-3\,dB}} = \frac{1}{\omega_t}$$

Here, $V_{step}$ is the size of the voltage step. With this exponential relationship, the time required for the first-order closed-loop circuit to settle with a specified accuracy can be found. For example, if 1% accuracy is required, then one must allow $e^{-t/\tau}$ to reach 0.01, which is achieved at a time of $t = 4.6\tau$. For settling to within 0.1% accuracy, the settling time needed becomes approximately $7\tau$. Also, note that just after the step input, the slope of the output will be at its maximum, given by

$$\left.\frac{d}{dt}v_{out}(t)\right|_{t=0} = \frac{V_{step}}{\tau} \qquad (5.31)$$

May be subject to slew rate

# Example 5.6 (page 216)

One phase of a switched-capacitor circuit is shown in Fig. 5.9, where the input signal can be modelled as a voltage step and 0.1% accuracy is needed in 0.1 μs. Assuming linear settling, find the required unity-gain frequency in terms of the capacitance values, $C_1$ and $C_2$. For $C_2 = 10C_1$, what is the necessary unity-gain frequency of the opamp? What unity-gain frequency is needed in the case $C_2 = 0.2C_1$?

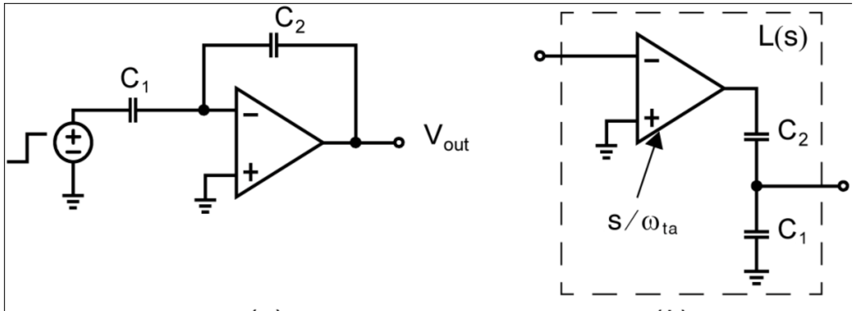
assume the opamp has a dominant pole response with unity gain frequency $\omega_{ta}$ (different from the unity gain frequency of $L(s)$, defined as $\omega_t$) and large dc gain. Hence, its frequency response at midband frequencies is well approximated by $\left(\frac{\omega_{ta}}{s}\right)$

$$L(s) \cong \left(\frac{\omega_{ta}}{s}\right)\left(\frac{C_2}{C_1 + C_2}\right) \qquad \omega_t = \omega_{ta} C_2 / (C_1 + C_2).$$

For $7\tau$ settling within 0.1 μs, we see that $\tau$ must be less than 14.2 ns. Since $\tau = 1/\omega_t$,

$$\omega_{ta} \geq \left(\frac{C_1 + C_2}{C_2}\right)\left(\frac{1}{14.2 \text{ ns}}\right)$$

For the case in which $C_2 = 10C_1$, a unity-gain frequency of $2\pi \cdot 12.3$ MHz is needed, whereas in the case of $C_2 = 0.2C_1$, $\omega_{ta}$ should be larger than $2\pi \cdot 66.8$ MHz.



(a) (b)

Chapter 5 Figure 09

The unconditional stability, predictable bandwidth, and predictable settling time of first-order systems are strong incentives to ensure feedback circuits have a dominant pole and, therefore, may be approximated by the single-pole model

$$L(s) = \frac{L_0}{(1 + s/\omega_{p1})}$$

# 5.3.2 Second order feedback systems

Although first-order feedback system is best, practical feedback amplifiers generally have more than one pole, as multiple single-stage amplifiers are usually needed to achieve a large DC gain.

A general second-order feedback loop transfer function is,

$$L(s) = \frac{L_0}{(1 + s/\omega_{p1})(1 + s/\omega_{eq})} \qquad (5.34)$$

It is assumed here that $w_{eq} > w_{p1}$.

A second order system is unconditionally (or absolutely) stable since a phase shift of -180 degrees is never reached. However, the Phase Margin may be so close to 0 depending on $L_0$, $w_{p1}$ and $w_{eq}$.

At frequencies much greater than the dominant pole frequency, $\omega \gg \omega_{p1}$, we see that $1 + j\omega/\omega_{p1} \cong j\omega/\omega_{p1}$, and so (5.34) can be accurately approximated by

$$(5.36) \qquad L(s) \cong \frac{L_0 w_{p1}}{s(1 + s/\omega_{eq})} \cong \frac{\beta\omega_{ta}}{s(1 + s/\omega_{eq})}$$

As $w_t = L_0 w_{p1} = A_0 \beta w_{p1} \cong w_{ta}\beta$

Dominant pole

Unity gain frequency of OpAmp

Valid for first order model

# 5.3.2 Second order feedback systems

The unity-gain frequency, $\omega_t$, can now be found by setting the magnitude of (5.36) equal to unity after substituting $s = j\omega_t$. Once this is done and the equation is rearranged, one can write,

$$\beta \frac{\omega_{ta}}{\omega_{eq}} = \frac{\omega_t}{\omega_{eq}} \sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2} \tag{5.37}$$

For the dominant pole special case in which the unity-gain frequency is much less than the equivalent nondominant pole frequency (i.e., $\omega_t \ll \omega_{eq}$), (5.37) may be simplified to

$$\omega_{ta} = \frac{\omega_t}{\beta} \sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2} \cong \frac{\omega_t}{\beta} \tag{5.38}$$

From (5.36), the phase shift, $\angle L(\omega)$, is found.

$$\angle L(\omega) = -90° - \tan^{-1}(\omega/\omega_{eq}) \tag{5.39}$$

This equation implies that at the unity-gain frequency, $\omega = \omega_t$, we have

$$PM = \angle L(\omega_t) - (-180°) = 90° - \tan^{-1}(\omega_t/\omega_{eq}) \tag{5.40}$$

and, therefore,

$$\begin{aligned} \omega_t/\omega_{eq} &= \tan(90° - PM) \\ \Rightarrow \omega_t &= \tan(90° - PM)\omega_{eq} \end{aligned} \tag{5.41}$$

# 5.3.2 Second order feedback systems

$$A_{CL}(s) = \frac{A_{CL0}}{1 + \frac{s(1/\omega_{p1} + 1/\omega_{eq})}{1 + L_0} + \frac{s^2}{(1 + L_0)(\omega_{p1}\omega_{eq})}}$$

$$H(s) = \frac{K\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} = \frac{K}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$

$$Q = \frac{\sqrt{(1 + L_0)/\omega_{p1}\omega_{eq}}}{1/\omega_{p1} + 1/\omega_{eq}} \cong \sqrt{\frac{L_0\omega_{p1}}{\omega_{eq}}} = \sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}} = \sqrt{\frac{\omega_t}{\omega_{eq}}\sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2}}$$

See (5.36) and (5.37)

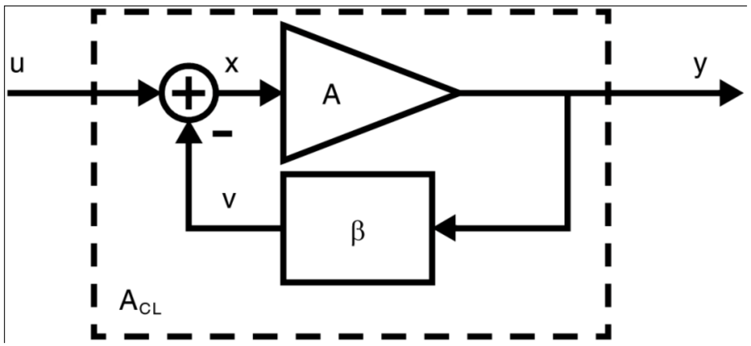
**Table 5.1** The relation ship betw een PM , $\omega_t/\omega_{eq}$, Q fact or, and percentage overshoot

| PM (Phase margin) | $\omega_t/\omega_{eq}$ | | Q factor | Percentage overshoot for a step input | |
|---|---|---|---|---|---|
| 55° | 0.700 | | 0.925 | 13.3% | |
| 60° | 0.580 | | 0.817 | 8.7% | |
| 65° | 0.470 | < | 0.717 | 4.7% | No freq. peaking |
| 70° | 0.360 | | 0.622 | 1.4% | |
| 75° | 0.270 | < | 0.527 | 0.008% | No overshoot |
| 80° | 0.175 | | 0.421 | - | |
| 85° | 0.087 | | 0.296 | - | |

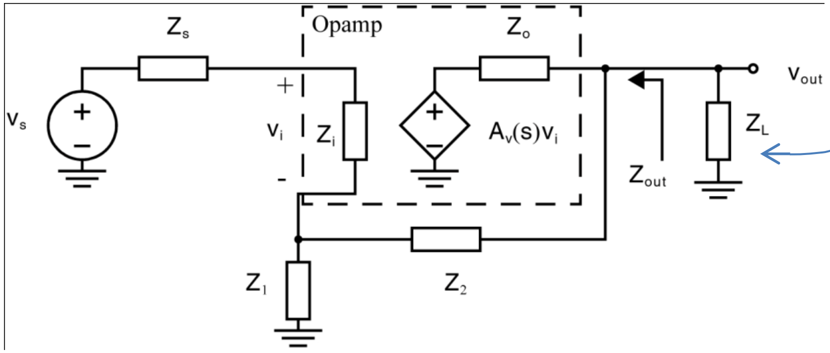
phase margin should be at least 75°, again, given both process and temperature variations. Hence, a nominal phase margin of 80° to 85° is often targeted to account for these variations.

# 5.4 Finding loop gain

From what we learned before, we can now analyze feedback amplifiers represented in small-signal models. In order to do so, we must cast the circuit of interest to the model below.




Chapter 5 Figure 01




Chapter 5 Figure 16

Unfortunately, this is often very difficult to do. The main difficulty is to identify the open-loop forward amplifier A and sometime feedback network is difficult to discern as well, especially some components are simultaneously involved in both forward amplifier and feedback network.

For the inverting/non-inverting amplifier, where the OpAmp is not ideal, it is tempting to conclude that A is simplify the voltage gain of the OpAmp, but when A=0, the close-loop output is not 0 (due to division of $V_s$, $Z_1$, $Z_2$ and $Z_o$, $Z_L$). In fact, the real A depends on not only OpAmp but also the above components.

# 5.4 Finding loop gain

Relatively speaking, find β is easier, as it is simply the inverse of the closed-loop gain.

Whereas there may be some ambiguity in defining A, the loop gain L is unique.

If L is known and β is taken by definition to be the reciprocal of the desired closed-loop gain, rather than trying to perform a rigorous analysis for A we can infer it from $A = L/\beta$. Therefore, our analysis of feedback circuits will be based on a knowledge of β, ours either by design or by inspection of a relatively simply circuit, and from a knowledge of L which can be estimated using a systematic procedure described in Section 5.4.1.

In addition, the closed-loop frequency response can be obtained using loop gain only without referencing to A as follows:

$$A_{CL}(s) = \frac{1}{\beta} \cdot \frac{L(s)}{1 + L(s)}$$

# 5.4.1 A generalized method to find loop gain

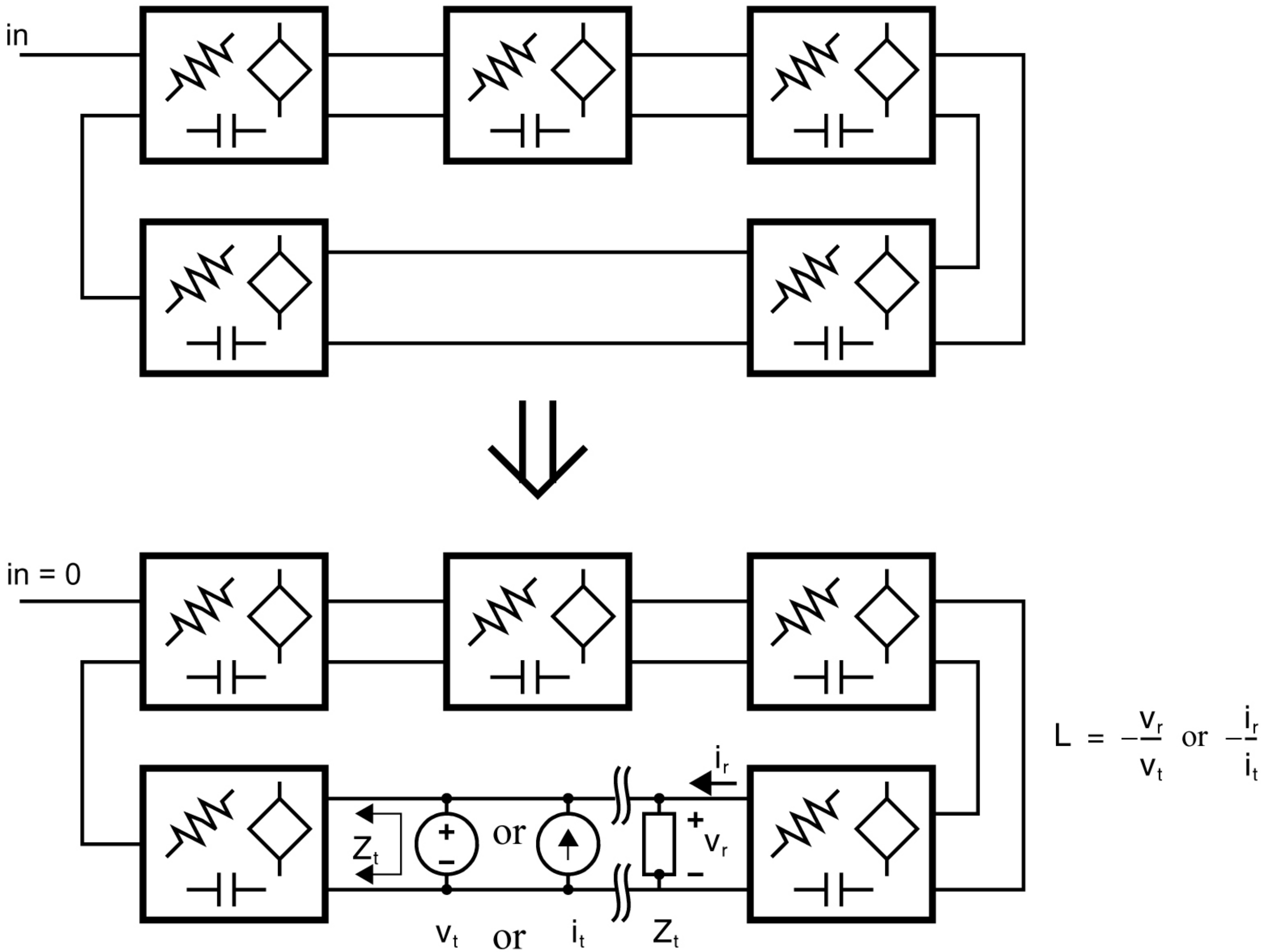
To find the loop gain of a feedback circuit, we break the loop, insert a test signal (either voltage or current) and see what signal the loop returns to the other side of the break. The procedure is illustrated in Fig. 5.11. Specifically, for any (small-signal) feedback circuit,

1. Set any independent sources in the circuit equal to zero, including the input source. Zeroing a voltage source means replacing it will a short circuit. (A short circuit is an ideal voltage source of 0 V.) Zeroing a current source means replacing it with an open circuit. (An open circuit is an ideal current source with a value of 0 A.)
2. Break the loop. Find the impedance at the break point, $Z_t$, and terminate the loop with this impedance as shown in Fig. 5.11.
3. Insert a test signal (either voltage, $v_t$, or current, $i_t$) into the loop at the break and find the returned signal: either the voltage across $Z_t$, $v_r$, or the current through $Z_t$, $i_r$. The loop gain is then

$$L = -\frac{v_r}{v_t} \text{ (voltage test signals) or } -\frac{i_r}{i_t} \text{ (current test signals).} \tag{5.53}$$

The key step is 2. Although the loop may be broken anywhere, it is convenient to do so right at an ideal voltage or current source so that the terminating impedance plays no role in the result, and hence may be omitted. Alternatively, one may chose a point in the loop where the terminating impedance is easily found.

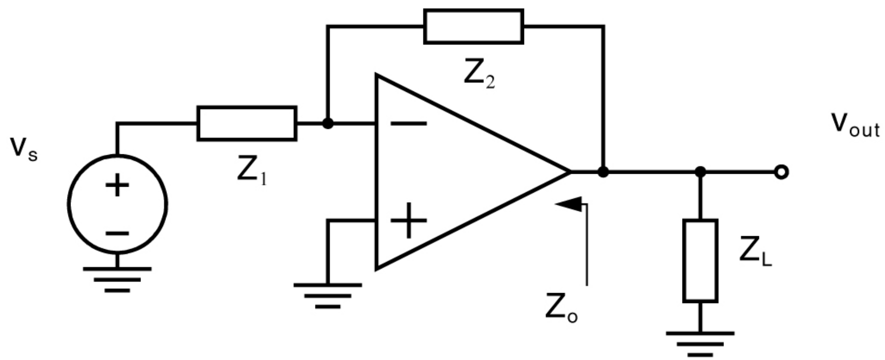
The loop gain obtained this way is an approximation as it neglects the possibility of signals circulating in the opposite direction around the loop. It is reasonable as long as the forward gain is much greater, which is generally the case for our feedback amplifiers to have large gain.




Chapter 5 Figure 11

# Example 5.8 (page 223)

Find the loop gain, $L(s)$, and closed-loop gain, $A_{CL}(s)$, of the inverting opamp configuration shown in Fig. 5.10 where the load impedance, $Z_L$, is infinite. Assume the opamp has a voltage gain $A_v$, and an infinite input impedance.




Chapter 5 Figure 10

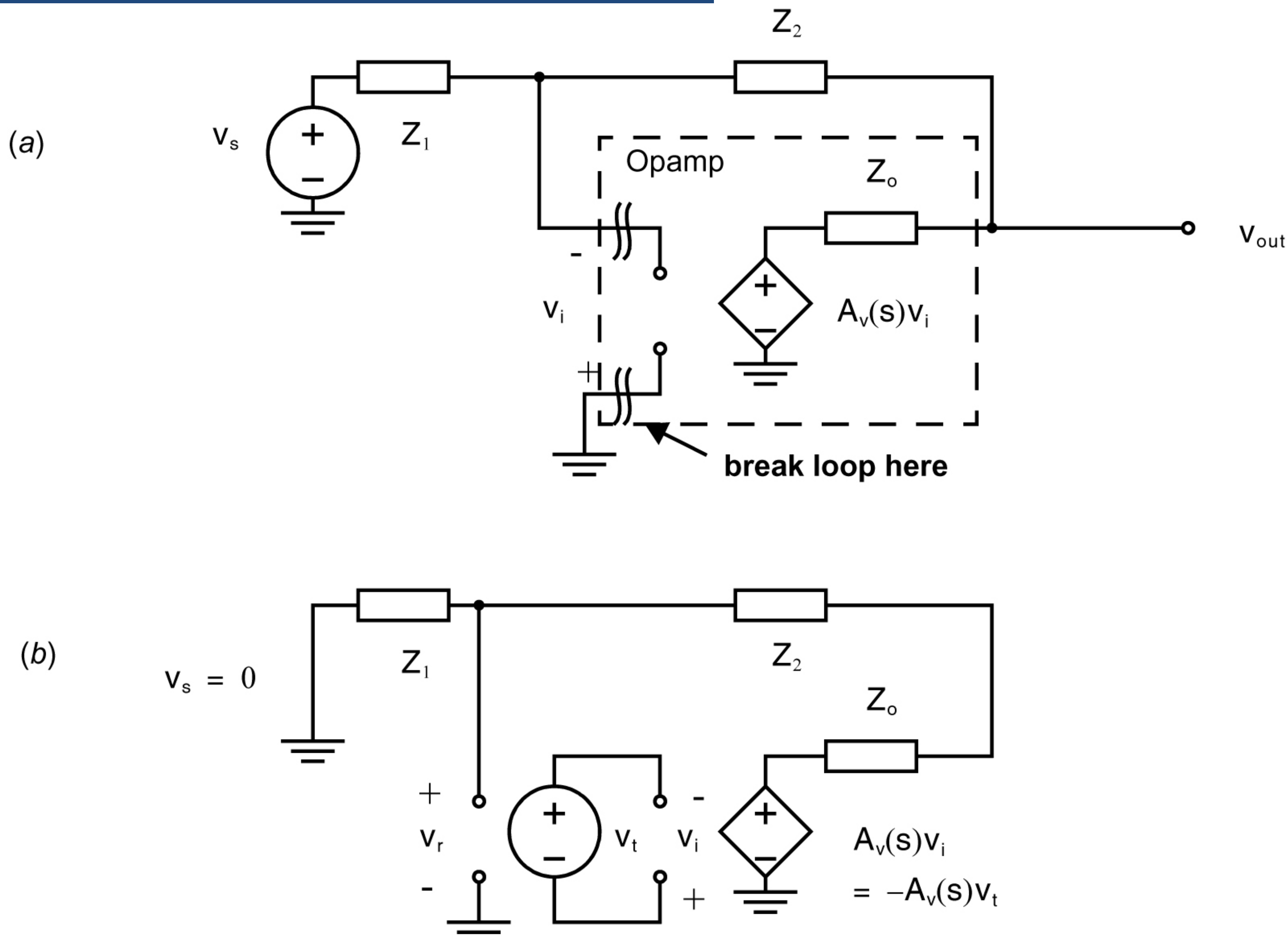
1. The input source is zeroed by setting $v_s$ to ground.
2. The loop is broken at the opamp input terminals. This point is chosen because the input impedance there is infinite, so finding $Z_t$ is trivial.
3. The test signal $v_t$ is injected into the new circuit, Fig. 5.12(*b*). The returned voltage is determined by nodal analysis,

$$v_r = -A_v(s)\frac{Z_1}{Z_1 + Z_2 + Z_o}v_t \qquad L(s) = -\frac{v_r}{v_t} = A_v(s)\frac{Z_1}{Z_1 + Z_2 + Z_o}$$

$$A_{CL}(s) = \frac{1}{\beta}\cdot\frac{L(s)}{1 + L(s)} = -\frac{Z_2}{Z_1}\cdot\frac{A_v(s)Z_1}{A_v(s)Z_1 + Z_1 + Z_2 + Z_o} \cong -\frac{Z_2}{Z_1}$$

As long as $A_v \gg (Z_1 + Z_2 + Z_o)/Z_1$,

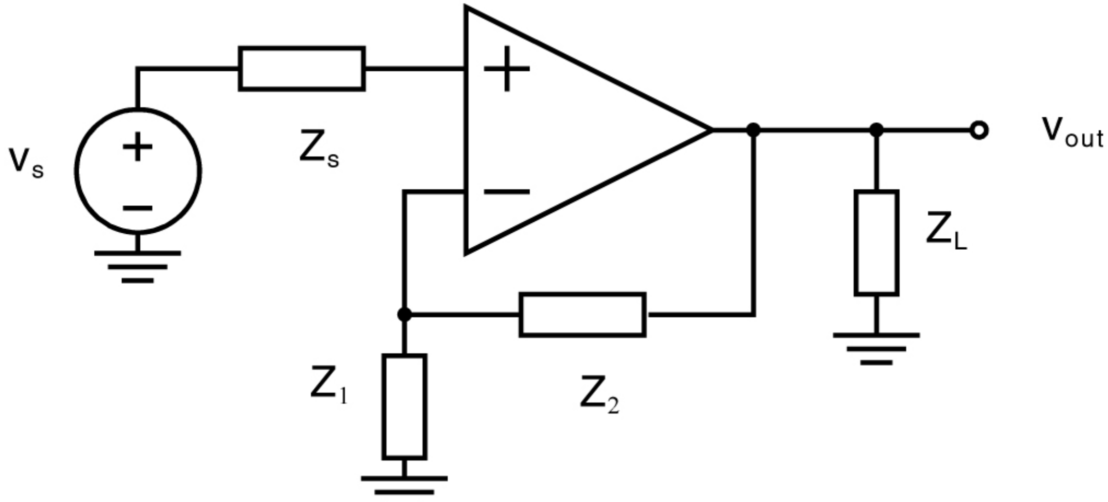
# Example 5.8 (page 223)




Chapter 5 Figure 12

# 5.4.2 Non-inverting amplifier

As in the inverting amplifier, the voltage gain of the OpAmp Av(s) is generally NOT to be considered as the open loop gain A(s) of the feedback amplifier, as the loading and non-idealities of the OpAmp all have effects, especially the loading.




Chapter 5 Figure 15

# Example 5.10 (page 227)

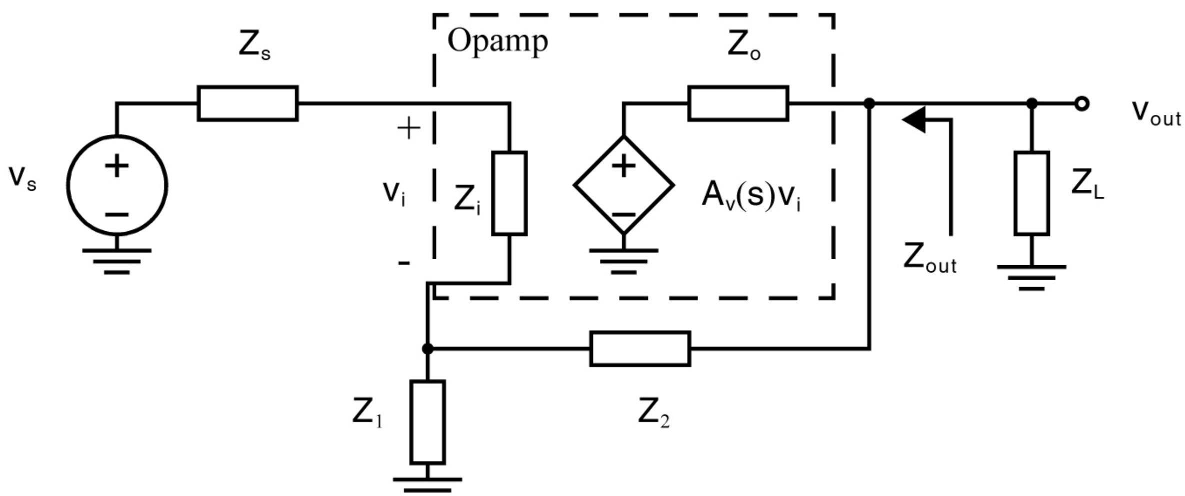
In order to determine the loop gain, we must analyze the circuit in Fig. 5.17. The loop is broken at the output of the controlled-source $A_v$, which models the opamp voltage gain. Since the termination impedance, $Z_t$, is inserted across an ideal voltage source, its value will have no effect on the analysis. The redrawn schematic at the bottom of Fig. 5.17 clearly shows

$$v_r = A_v(s)v_i = -A_v(s)\frac{Z_i}{Z_o}\left[\frac{Z_L||Z_o}{Z_L||Z_o + Z_2 + Z_1||(Z_s + Z_i)}\right]\left[\frac{Z_1}{Z_s + Z_i + Z_1}\right]v_t$$

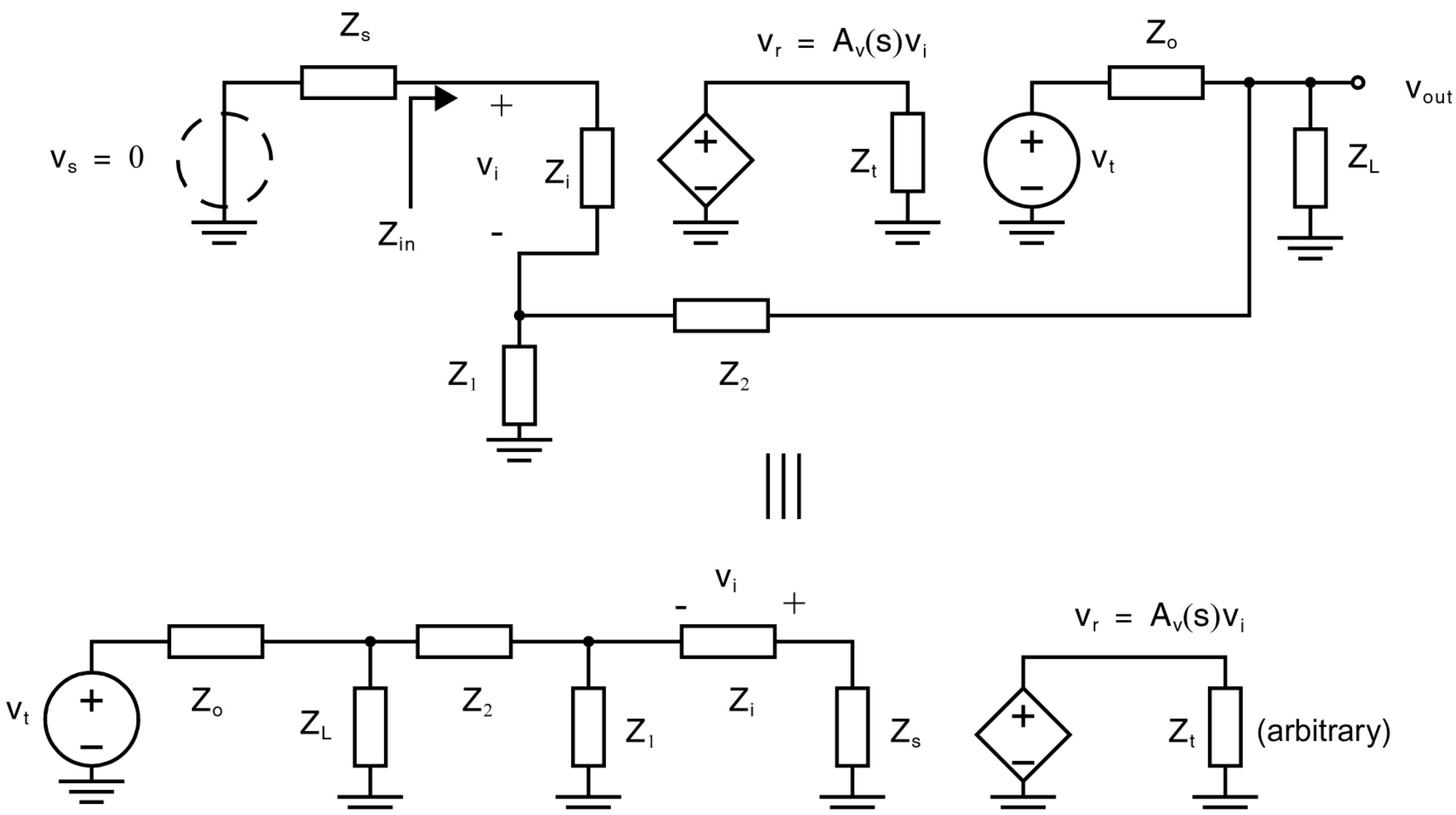
$$L(s) = A_v(s)\frac{Z_i}{Z_o}\left[\frac{Z_L||Z_o}{Z_L||Z_o + Z_2 + Z_1||(Z_s + Z_i)}\right]\left[\frac{Z_1}{Z_s + Z_i + Z_1}\right] \quad (5.58)$$

$$L(s) \approx A_v(s)\frac{Z_1}{Z_1 + Z_2} = A_v(s)\beta$$

If $Z_i \gg Z_1, Z_s$ and $Z_o \ll Z_L, Z_2$

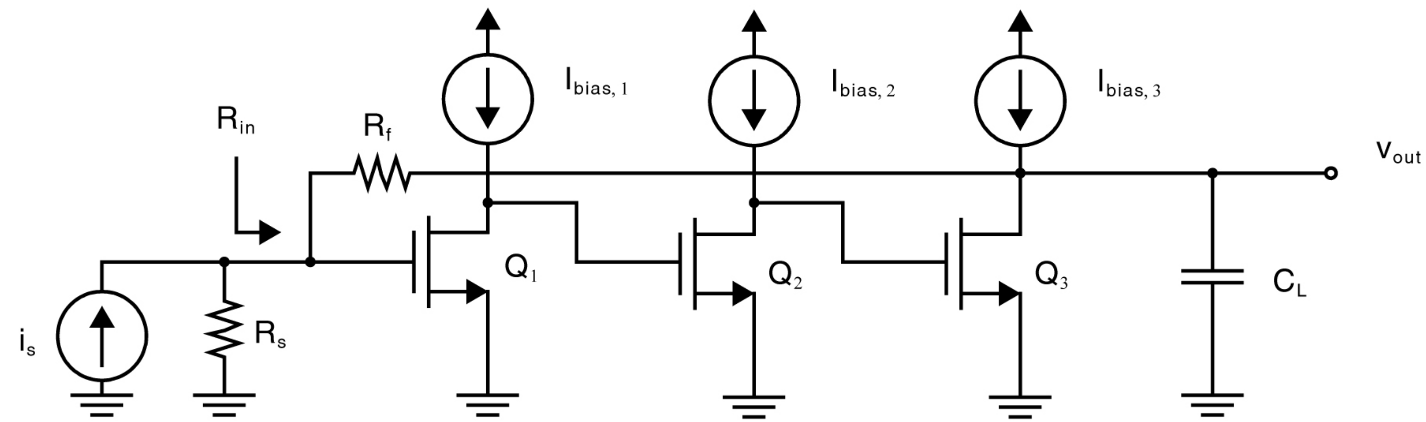



Chapter 5 Figure 16

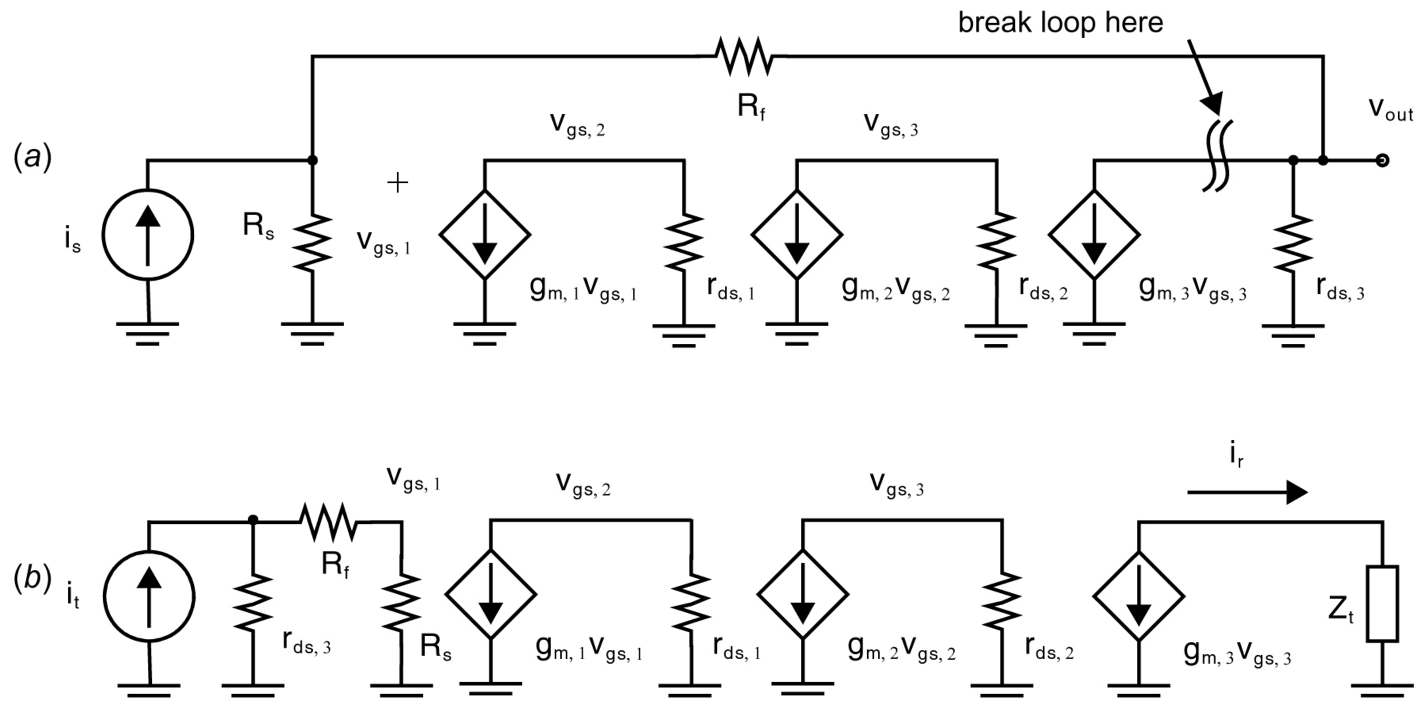



Chapter 5 Figure 17

# Example 5.14 (page 233)

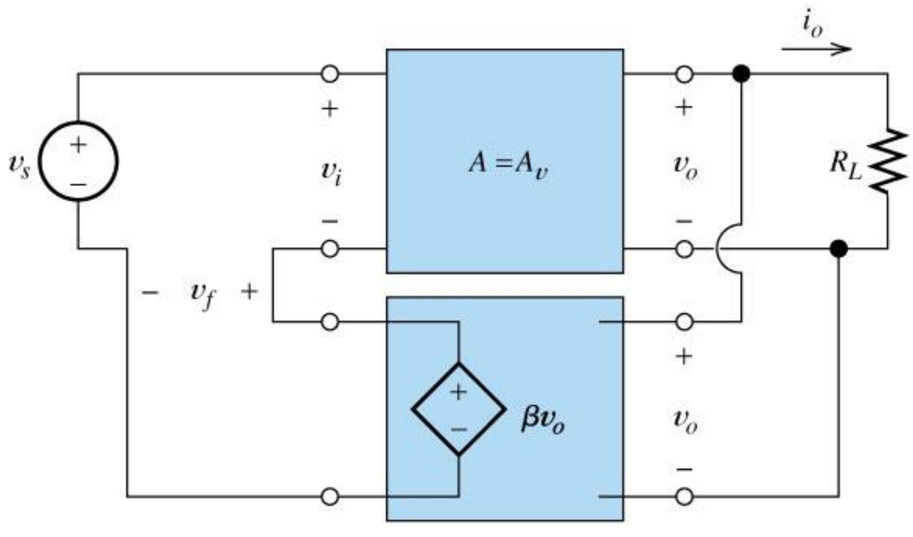



Chapter 5 Figure 22

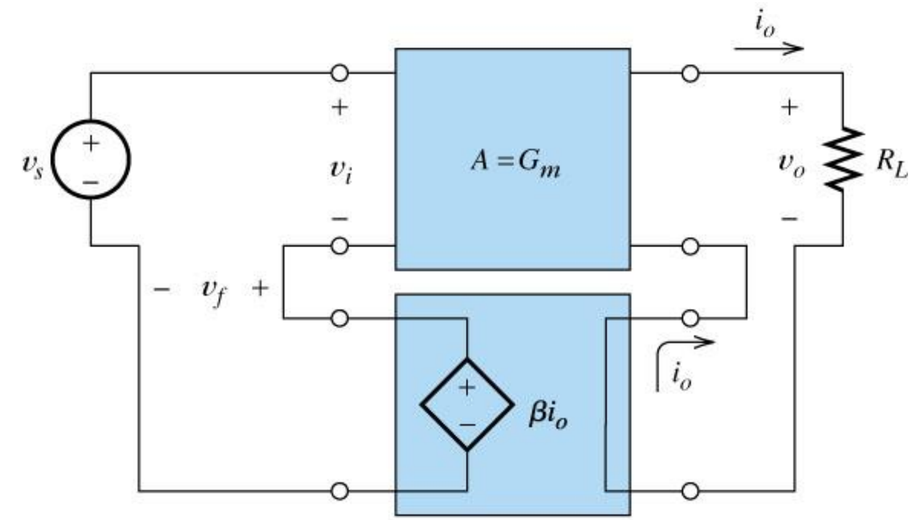



Chapter 5 Figure 23

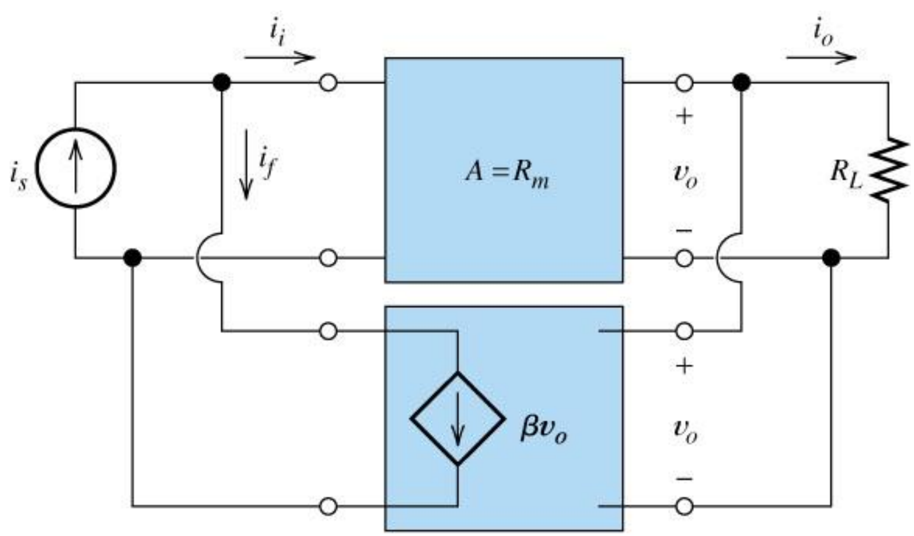
# Amplifier negative feedback types



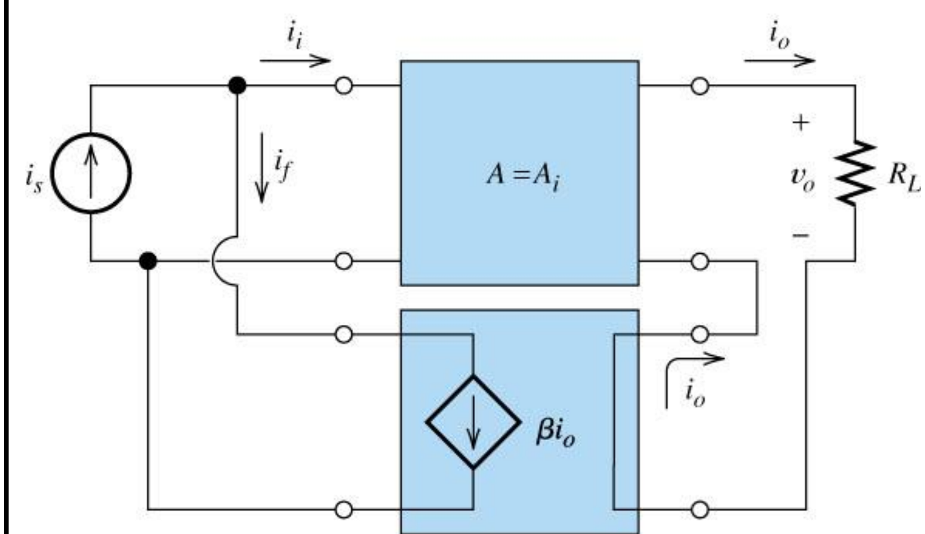

(a) Series voltage feedback




(b) Series current feedback



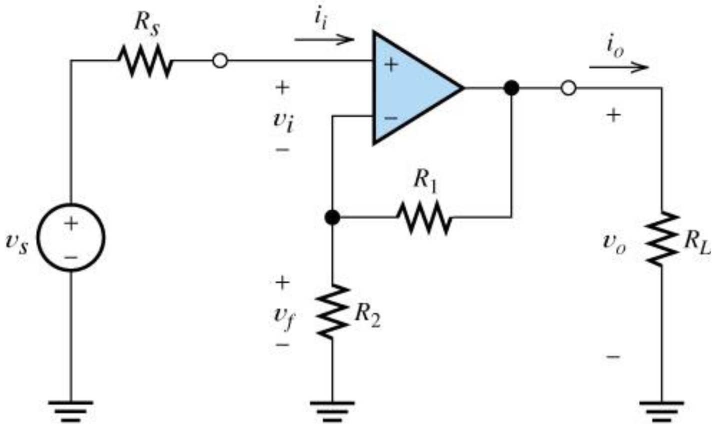

(c) Parallel voltage feedback



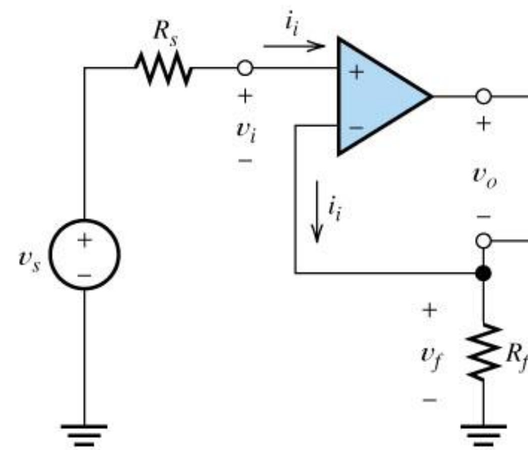

(d) Parallel current feedback

# Some practical feedback network in amplifiers

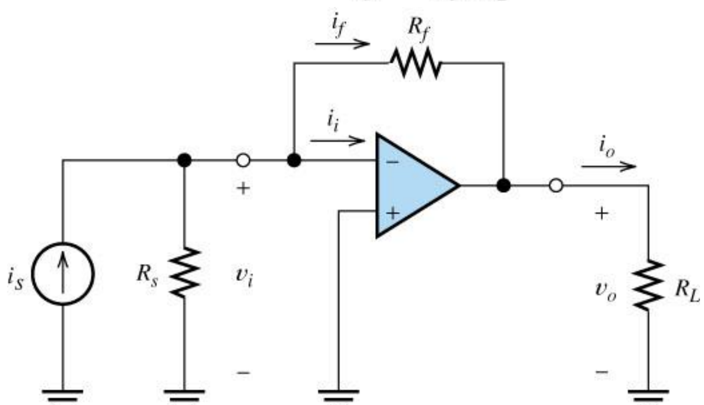
- In practice, negative feedback network consists of resistor or capacitors , whose value is much more precise and stable than active devices (such as transistors). Then amplifier characteristics mainly depends on feedback network, thereby achieving precision and stability.



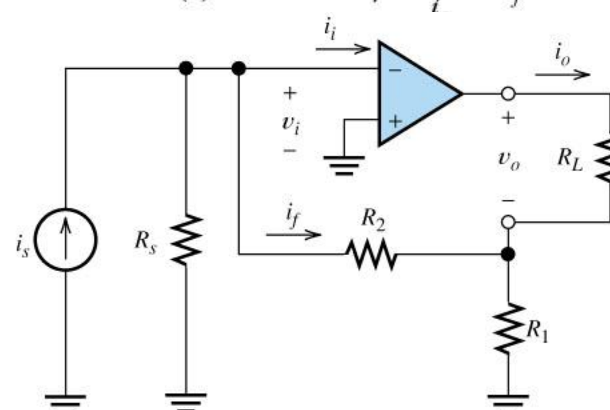

(a) Series voltage $\beta = \frac{v_f}{v_o} = \frac{R_2}{R_1 + R_2}$




(b) Series current $\beta = \frac{v_f}{i} = R_f$

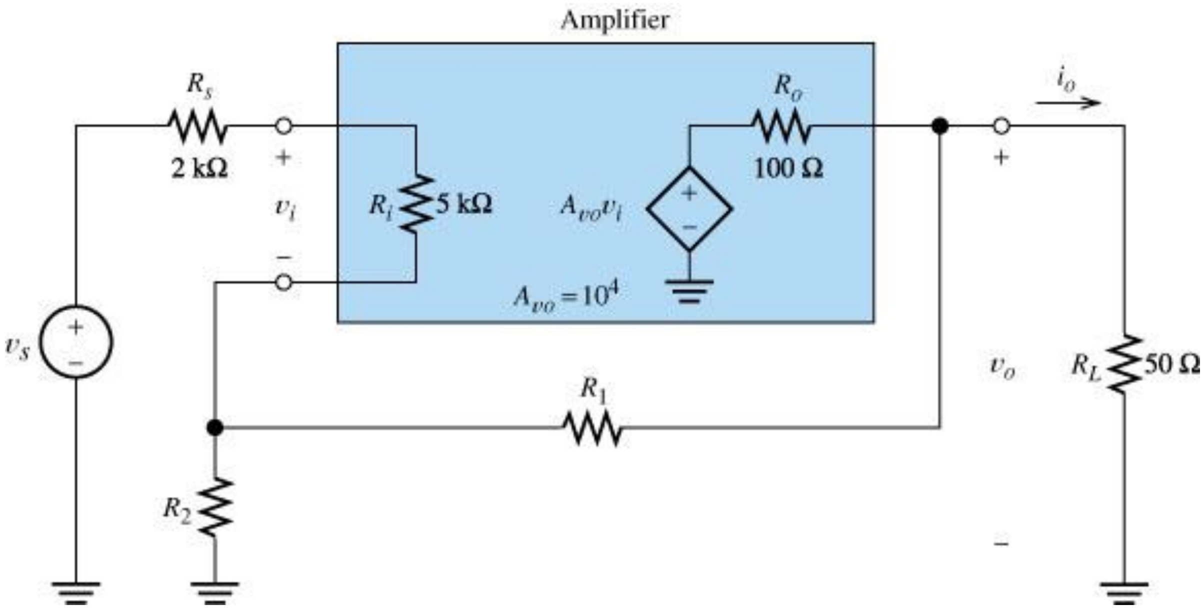


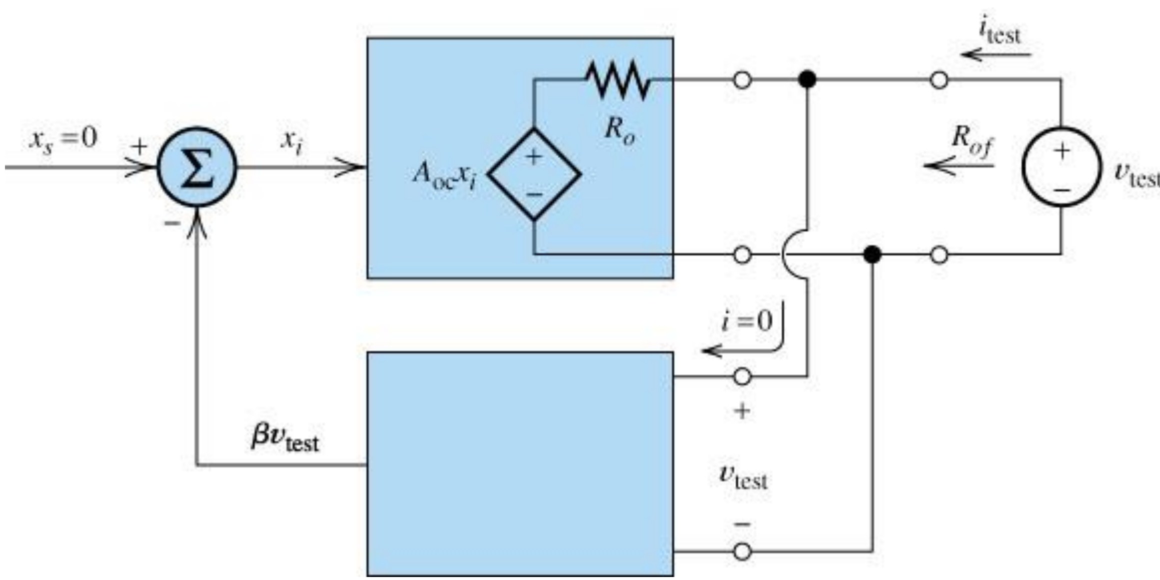

(c) Parallel voltage $\beta = \frac{i_f}{v_o} = -\frac{1}{R_f}$




(d) Parallel current $\beta = \frac{i_f}{i_o} = -\frac{R_1}{R_1 + R_2}$

# An example of feedback voltage amplifier




- Real input and output impedance is different from what is predicted from the formula in the ideal case. But it is always a good initial guess.
- You might need to try out multiple iterations to achieve a good design.