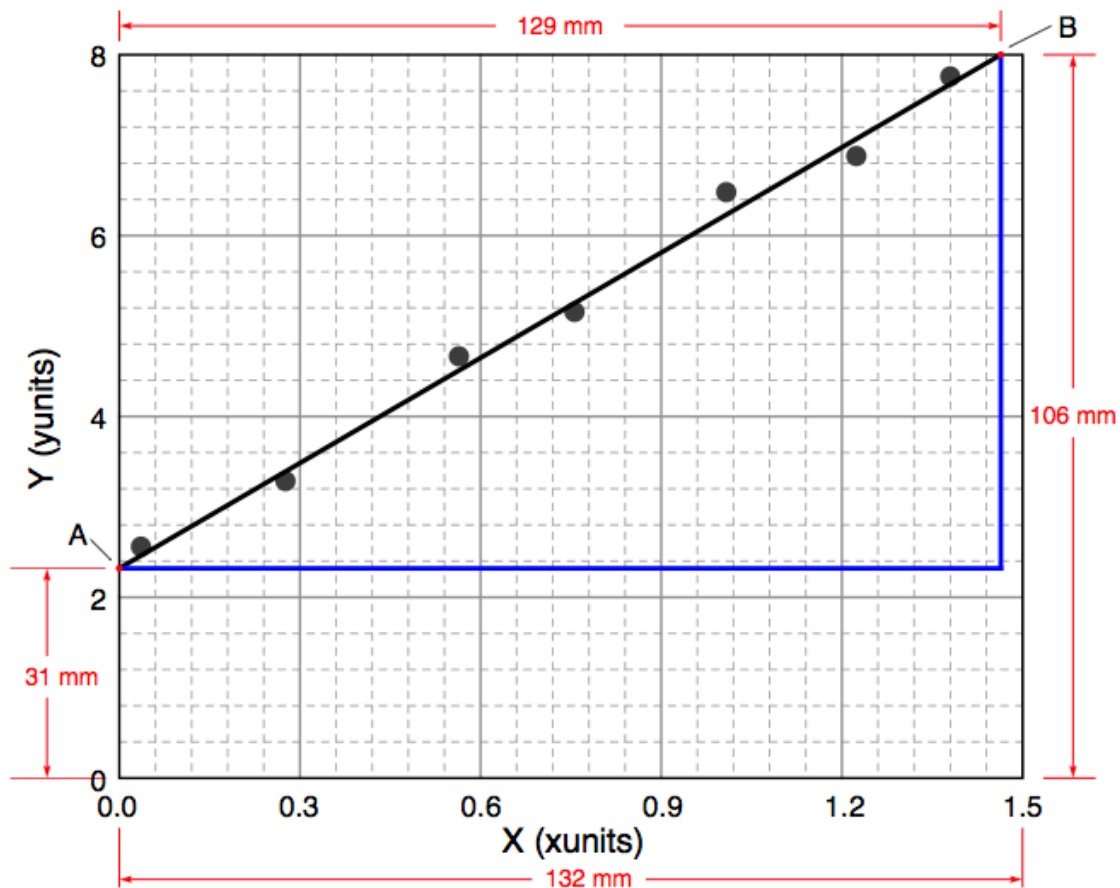


Finding slopes accurately. When calculating the slope of a line fitting the data, you must never use specific data points in the calculation. Instead, you should extend your line to the boundaries of your grid and use the two points where the line exits the grid. Using two widely separated points will allow a more precise estimate of the slope. Computer-generated graphs often have widely-spaced grid lines (if any at all) that make it difficult to read off these coordinates with much precision. When the printed grid lines are too widely spaced, you need to do a little more work to find the coordinates. This involves measuring distances on the graph with a ruler and finding the scale factors that relate distances to the units used in the measurement. This is illustrated in the following graph and discussion.



An example is shown of data that seem to follow a straight line, $Y = mX + b$. After plotting the data and printing the graph, a straight line was drawn by hand to best represent the linear trend of the data. Finding the Y-intercept and the slope of the line is the usual analysis required. The line crosses the Y-axis at point A. The Y-coordinate of point A is the Y-intercept, b . To find the slope, m , we use the triangle formed by our best straight line connecting points A and B and the two horizontal and vertical lines shown:

$$m = \frac{\Delta Y}{\Delta X} = \frac{Y_B - Y_A}{X_B - X_A}$$

Two of the four coordinates we need are easily found on the graph: $X_A = 0.00$ xunits and $Y_B = 8.00$ yunits. Determining Y_A and X_B will require more effort. Four distances need to be measured with a ruler on the printed graph:

- [1] the total length of the Y-axis (106 mm),
- [2] the total length of the X-axis (132 mm),
- [3] the vertical distance of A from the X-axis (31 mm), and
- [4] the horizontal distance to B from the Y-axis (129 mm).

(You may measure distances different from those shown as a result of scale changes in the printing process. If you repeat the calculations below with your own distances, you should get practically the same slope and intercept values. This is a good way to check your understanding.)

The coordinates Y_A and X_B can be found from proportionalities:

$$\frac{Y_A}{8.00 \text{ yunits}} = \frac{31. \text{ mm}}{106. \text{ mm}} \Rightarrow Y_A = \frac{31. \text{ mm}}{106. \text{ mm}} 8.00 \text{ yunits} = 2.340 \text{ yunits}$$

$$\frac{X_B}{1.50 \text{ xunits}} = \frac{129. \text{ mm}}{132. \text{ mm}} \Rightarrow X_B = \frac{129. \text{ mm}}{132. \text{ mm}} 1.50 \text{ xunits} = 1.465 \text{ xunits}$$

More significant figures than are realistic have been retained for intermediate calculations. The Y-intercept is then known: $b = Y_A = 2.34$ yunits, and the slope can be calculated:

$$m = \frac{Y_B - Y_A}{X_B - X_A} = \frac{(8.00 - 2.34) \text{ yunits}}{(1.465 - 0.00) \text{ xunits}} = 3.86 \frac{\text{yunits}}{\text{xunits}}$$

The equation of the line is then $Y(X) = 3.86 \frac{\text{yunits}}{\text{xunits}} X + 2.34 \text{ yunits}$. Given the amount of scatter in this data, reporting the slope and intercept to 3 significant figures is probably unrealistic. Rounding each to two figures is a reasonable thing to do: $Y(X) = 3.9 \frac{\text{yunits}}{\text{xunits}} X + 2.3 \text{ yunits}$. If the data all fell xunits much closer to the line, another significant figure in the final results could be appropriate.

Further cautionary notes. If your printed graph does not include the origin

(0,0) at the lower left, you must take that into account in calculating the true X and Y coordinates from the measured distances on paper and exercise greater caution in finding the true Y-intercept. If the left side of your grid is not the Y-axis (the line representing $X = 0$), the point A is *not* the Y-intercept!

Sometimes the line you draw may have a negative slope or the line may exit the grid at the top and bottom edges. Calculating the slope is still straight forward, just be sure that point B is to the right of point A: $X_B > X_A$. Finding the correct Y-intercept will again be tricky. That's why it's usually wise to carefully adjust the ranges of your X and Y axes to include the origin on your graph. You should have a very good reason to do otherwise!