Dynamical Systems
Math 5260
Lab Tasks:
Tangent (saddle-node) and period-doubling Bifurcations for $x^2 + c$

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Many periodic orbits are "born" As c decreases in the quadratic family x^2+c, a host of periodic orbits are "born" through either saddle-node bifurcations or period doubling bifurcations.

Do the following for period-q orbits for each of $q = 1, 2, ..., 4$:

By plotting the qth iterate of the quadratic family maps $x^2 + c$ for c values decreasing from 1 to -2, determine approximate parameter values where fixed points for the qth iterate are born. Determine whether the births are due to a saddle-node or a period-doubling bifurcation. Determine the prime period of the orbit being born.

Suggestion: Use a combination of Nonlinear Web (BU Website) and Mathematica. Nonlinear Web has the big advantage of a slider to change the value of c, so you can quickly get an idea of where the bifurcations are, but it is hard to identify the bifurcation parameter values with much accuracy. Mathematica has the advantages of being able to plot multiple graphs at once, specifying parameter value with high precision, and specifying plot bounds to zoom in on a particular region. High precision parameter specification, and zooming in on the graphs enables a more accurate estimate of the bifurcation parameters. Multiple graphs allow us, for example, to plot both the first and second iterates to see which fixed points of the second iterate are prime period 1 and which are prime period 2.

(Try Plot[f[x,c], f[f[x,c],c], x,-2,2] where f[x,c] :=$x^2 + c$ and c is a fixed parameter value.)