MIDTERM TOPIC LIST
Dynamical Systems
Math 5260
Bruce Peckham
October 14, 2007
For midterm on Fri. Oct. 26, 2007: 8:30-9:50

In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on “basic” material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

1. Definitions to know:
   - Fixed point, periodic point, orbit, cycle, period, prime or least period
   - Attracting, superattracting, repelling, neutral periodic point (orbit, cycle); use of chain rule in determining these adjectives
   - Phase portrait
   - Graphical analysis
   - Discrete dynamical system vs. continuous dynamical system
   - Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
   - The sequence space Σ, the “usual” metric on Σ, the shift map σ.
   - Three properties of a chaotic system and all terminology used in the def. of the properties.
   - A *dense* in B for $A \subset B$.
   - Homeomorphism, topological conjugacy, topological semiconjugacy
2. Results to know:
   - Relationship between the shift map on the symbol sequence space and the quatratic map (for small enough c) on the invariant Cantor set.
   - Sarkovkii’s theorem, including Sarkovskii’s ordering
   - “Negative Schwarzian Derivative Property” (which is true for any quadratic): Any attracting periodic orbit must attract a critical point.

3. Techniques to know:

    - Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
    - Locating period-n pts/orbits of f from graphs of f and f^n.
    - Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
    - Determining stability of periodic orbits either analytically or from graphs.
    - Constructing a graph of f so that f has, for example, a periodic orbit of a certain period and certain derivative (of f^n).
    - Constructing the graph of iterates of f given the graph of f.
    - The construction of the invariant Cantor set Λ for $x^2 + c$ with c small enough.
    - The construction of an itinerary map.
    - Given a map and a change of variables, find the equation for the map in the new variables.
    - Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
    - Determining the number of prime periodic orbits of each period for $x^2 - 2$ (equivalently $2x \pmod 1$, $4x(1-x)$, σ)
    - Determining the number of period-n windows for each n in the orbit diagram for the family $x^2 + c$.
    - Locating each period-n window in the orbit diagram for the family $x^2 + c$.

4. Proofs to know:

    - f continuous, I closed interval, $f(I) \subseteq I$ or $f(I) \supseteq I$ implies there is a fixed point for f in I.
    - f continuous, f has a periodic point implies f has a fixed point
    - Prove $\sigma : \Sigma \to \Sigma$ is chaotic. (Prove any or all 3 properties.)

5. Anything else we've covered that I think is easy.