Math 3280: DE+LA. Test 3 partial answers. Prof. Bruce Peckham

1. $y_c(x) = C, y_p(x) = \frac{x^3}{3}$
2. $y_p(x) = \frac{2}{7}e^{4x}$
3. $y(x) = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + c_4 \cos(x) + c_5 \sin(x) + c_6 x \cos(x) + c_7 x \sin(x)$
4. $D - 3$
5. $y(x) = c_1 e^{3x} + c_2 \cos(2x) + c_3 \sin(2x)$
6. $G(s) = 3\frac{2}{s^3} - 5\frac{s-4}{(s-4)^2+3^2}$
7. proof in book
8. $g(t) = u(t-3)(t-3)^2$; $G(s) = e^{-3s}\frac{2}{s^3}$
9. $y(t) = 3e^{-5t}$
10. $Y(s) = \frac{1}{s^2-3s+1}(\frac{3s}{s^2+16} + 2s - 6)$
11. $f(t) = 3e^{-t}\cos(6t) - \frac{4}{3}e^{-t}\sin(6t)$
12. $y_p(t) = At\cos(2t) + Bt\sin(2t)$
13. $y' = v, v' = 2y - v$; $\vec{x}' = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} \vec{x}$, where $\vec{x} = \begin{pmatrix} y \\ v \end{pmatrix}$.
14. Yes. (plug and check)
15. Other eigenvalue and eigenvector: 2 and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Solution: $\vec{x}(t) = c_1 e^{-3t}\begin{pmatrix} 4 \\ 1 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (or equivalent)
16. (a) $(0,0)$ and $(0,1)$.
    (b) $< -\frac{1}{2}, -2 >$
17. (Extra Credit) $\vec{x}(t) = c_1 e^{2t}\begin{pmatrix} \cos(4t) \\ \sin(4t) \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} \sin(4t) \\ -\cos(4t) \end{pmatrix}$