Math 3280, Differential Equations with Linear Algebra
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Brief Topic Summary for Test 2

Differential Equations:

1. Analytic Solutions (Sections 5.1, 5.2)
   (a) Structure of general solution to $y'+p(x)y=q(x)$: $y(x)=y_c(x)+y_p(x)=c_1y_1(x)+y_p(x)$.
   (b) Structure of general solution to $y''+p(x)y'+q(x)y=r(x)$: $y(x)=y_c(x)+y_p(x)=c_1y_1(x)+c_2y_2(x)+y_p(x)$.
   (c) Finding $y_1(x)$ and $y_2(x)$ for $ay''+by'+cy=0$ (constant coefficient linear, homogeneous - if the characteristic polynomial has real distinct or real repeated roots).
   (d) Using initial conditions, solve for c_1 and c_2 for second order linear differential equations.
   (e) Linear independence of y_1 and y_2. Use of Wronskian as a shortcut. More generally, linear independence of n functions.
2. Qualitative Solutions (labs - not on test)
3. Numerical Solutions (labs - not on test)
   (a) ANY!!!!: Euler's method (Runge-Kutta, ...)
4. Models (labs - one question might be on test)
   (a) Exponential growth (population), decay (radioactive decay)
   (b) Heating/Cooling
   (c) Falling object/parachute
   (d) Mixing
   (e) Logistic population growth

Linear Algebra (Secs 3.1-3.6, 4.1-4.4, 4.7)

1. Solve $Ax=b$ (Row reduction, row echelon form, reduced row echelon form and interpretation of row reduced matrices for no solution, unique solution, infinity of solutions, number of free parameters)
2. For A an $n\times n$ matrix: Compute Det(A), A^{-1} using row reduction
3. if Det$(A)\neq 0$, A^{-1} exists and there is a unique solution to $A\mathbf{x}=\mathbf{b}$
4. Vector Space/subspace, basis, linearly independent/dependent - including formal equations that must be satisfied to be dependent, span - including formal equations that must be satisfied for a given set of vectors to to span a (sub)space, dimension
5. Vector space examples: $\Re^2, \Re^3, \Re^n, M_{m\times n}, F$.
6. Vector subspace examples:
   (a) in $\Re^n$: origin, lines through origin, (hyper)planes through the origin, solutions to $A\mathbf{x}=\mathbf{0}$.
   (b) in F: solutions to $y'+p(x)y=0$ (1 dimensional) or $y''+p(x)y'+q(x)y=0$ (2 dimensional).
   (c) in $M_{n\times n}$: Diagonal matrices, ...
   (d) in any vector space: linear combinations of any set of vectors
   (e) Proof of a subset being a subspace (closed under vector addition and scalar multiplication) vs. example showing a subset is not a subspace
   (f) Shortcuts if you know the dimension of a vector (sub)space is n: any set of more that n vectors must be linearly dependent; no set of fewer than n vecors can span the space.