Math 3280: DE+LA. Test 2 partial answers. Prof. Bruce Peckham

1. (a) $y(x) = c_1 e^x + c_2 e^{-4x}$
   (b) $y(x) = c_1 e^{2x} + c_2 x e^{2x}$

2. $y(x) = \frac{7}{4}e^{2x} + \frac{5}{4}e^{-2x} - 2$

3. (a) 63
   (b) 43

4. 32

5. $\begin{pmatrix} 1 & 3 & 8 \\ 2 & -1 & 1 \\ 3 & 0 & -2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$

6. $(x_1, x_2) = (-9, 11)$

7. $\left\{ t \begin{pmatrix} 5 \\ -2 \\ 1 \\ 1 \end{pmatrix} \right\}$ (t is any real number))

8. Basis: $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right\}$ (or any two of the three given vectors, or any two independent vectors in $\Re^2$, since the span of the three given vectors is all of $\Re^2$.

9. (a) $A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$
   (b) Leave row 1 unchanged; replace row 2 with itself - 2 times row 1; multiply row three by 3;

10. (a) False. The zero vector is not a solution. (b) False. Need three vectors in a basis for $\Re^3$.

11. Solve the system $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ for c_1 and c_2 using, for example, row reduction. It should have the unique solution $c_1 = 0, c_2 = 0$. This means the two vectors are linearly independent.

12. There are several methods, but the easiest is to compute the Wronskian determinant of $\{e^x, e^{2x}, e^{3x}\}$. It should be $2e^{6x}$. Since this is not identically zero (it is never zero), the three functions are linearly independent.

13. Solve the system $c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ for c_1 and c_2 using, for example, row reduction.

    It should turn out that there is no solution. Therefore, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is not in the span of the other two vectors.

14. You must show T is closed under vector addition and scalar multiplication:
   (a) Let $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix} \in T$. This means $a + 2b = 0$ and $c + 2d = 0$. Then $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$. Since $(a+c)+2(b+d) = (a+2b)+(c+2d) = 0+0 = 0$, then $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} \in T$. So T is closed under vector addition.
   (b) Let $\begin{pmatrix} a \\ b \end{pmatrix} \in T$. This means $a + 2b = 0$. Let $c \in \Re$. Then $c\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ca \\ cb \end{pmatrix}$. Since $(ca) + 2(cb) = c(a + 2b) = c0 = 0$, then $c\begin{pmatrix} a \\ b \end{pmatrix} \in T$. So T is closed under scalar multiplication.