Chapter 6: Demand

Introduction
- Overview
- Comparative Statics
- Endogenous & Exogenous Variables

Overview
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- Review
- The Nature of Commodities & Graphical Representation

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Cross-Price Changes

Inverse Demand

Summary
- Chpt 6: Categories and definitions.
- Chpt 8: Price change → demand.
- Chpt 14: Price change → welfare.

Discussion: Situational Change & Consumer’s Reaction

- marketplace.publicradio.org/display/web/2008/06/09/charities/
- With the tools we have learned so far, propose a way to measure or predict "a big dent in volunteerism" triggered by the increasing gas price.
It is necessary (and critical) to observe the change in demand $x^*$ (endogenous) caused by given parameters $p$ and $m$ (exogenous).

Comparative Statics

Comparative statics analysis compares the values of endogenous variables at the different values of the exogenous variable while other parameters held constant.

ceteris paribus.

Why can’t we move multiple parameters at the same time like in the real world?

Prediction becomes less clear-cut (and confusing):

1. $p_C: 2 \nearrow 5$ ceteris paribus $\Rightarrow x_C^*: 3 \searrow 2$.
2. $p_C: 2 \nearrow 5$ while $p_T: 1 \nearrow 3$ $\Rightarrow x_C^*: 3 \rightarrow$ indefinite.

Endogenous & Exogenous Variables

- Endogenous & exogenous variables:
  - Given $\succeq$, $p$ and $m$, UMP predicts which $x^*$ Greg will choose.
  - UMP does not predict the movement of $p$, $m$ and change in tastes $\succeq$. 

![Diagram](image_url)
**Marshalian Demand Function**

**Marshalian demand function** returns the optimal bundle at each given price and income, denoted by

\[
\phi(p_1, p_2, m) = (\phi^1(p_1, p_2, m), \phi^2(p_1, p_2, m)).
\]
Change in Parameters

- Change in price causes rotation to the budget constraint.
- Change in income causes parallel shifts to the budget constraint.

Lots of definitions to learn today to describe and classify different types of commodities.
The Nature of Commodities & Graphical Representation

\[ \uparrow \phi^1(p, m) \quad \downarrow \phi^1(p, m) \]

\[ \uparrow m \quad \text{normal income-inferior} \]

\[ \downarrow p_1 \quad \text{LOD Giffen} \]

\[ \uparrow p_2 \quad \text{substitute complement} \]

\[ m \quad \text{income expansion path} \]

\[ p_1 \quad \text{price offer curve} \]

\[ x_1 - x_2 \quad \text{on } x_1 - \text{parameter} \]
A commodity is called a **normal** good if increase in income leads to increase in consumption **while other things being equal**. Otherwise, a commodity is **income-inferior**.

- Does an income-inferior good really exist?
Mass-produced cheesecakes (2 days past the expiration date) & fresh home-made cheesecakes
Income Expansion Path

Income expansion path represents the collection of optimal bundles on $x_1 - x_2$ plane under varying income levels while $p = (p_1, p_2)$ is held constant.

Engel Curve

A plot of quantity demanded against income, with fixed $p = (p_1, p_2)$ is called an Engel curve.
Cobb-Douglas Utility Function

\[
\max_{x_C, x_T} u(x_C, x_T) = x_C x_T^2 \quad \text{s.t.} \quad x_C + x_T = m.
\]

- MRT at \((x_C, x_T)\) is \(\frac{x_C}{2x_T}\).

\[
\phi(m, p = (1, 1)) = (\phi^1(m, p), \phi^2(m, p)) = \left(\frac{m}{3}, \frac{2m}{3}\right).
\]
- Engel curve for cheesecakes at \(p = (1, 1)\): \(m = 3x_C^*\).
Example 1: Cobb-Douglas

**Income Expansion Path**

Indifference curves for different income levels (m=3, m=6, m=9).

**Engel Curve**

The Engel curve showing the relationship between income (E) and expenditure on a particular good (x_C).
Consider a bundle of coins $x_C$ and tea $x_T$.

**Quasi-Linear Preferences**

$$\max_{x_C, x_T} u(x_C, x_T) = x_C + \sqrt{x_T} \quad \text{s.t.} \quad x_C + .5x_T = m.$$ 

- MRS at $(x_C, x_T)$ is $-2\sqrt{x_T}$.
- $\phi(m, p = (1, .5)) = (m - 1, 2)$.
- Recall MRS is same at each $x_T$ regardless of the amount of $x_C$. 

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**Figure: Income Expansion Path**

- Indifference curves for $m = 2$, $m = 4$, and $m = 6$.
Example 2: Quasi-Linear Preferences

Engel Curve for Coins

Engel Curve for Tea
Example 3: Perfect Substitutes

- Consider a bundle of six-packs and bottles of Corona \((x_6, x_1)\).
- Suppose \(p = (p_6, p_1) = (10, 2)\).
- MRS is \(-6\) while the relative price is \(-5\).
Example 3: Perfect Substitutes

Engel Curve for Six–Packs

Engel Curve for Bottles
Example 4: Perfect Compliments

Discussion

Consider a bundle (cereal, milk) = (xC, xM).

- Greg says he can’t have cereals without milk and the only time he has milk is when he eats his cereals.
- Greg’s preferred cereal-milk ratio is 1 to 1.
- \((p_C, p_M) = (2, 2)\).
- What do Greg’s Engel curves look like? (curve? flat?)
- Consider \(m = 4\) and \(m = 8\) for example.

Income Expansion Path

Figure:
Example 4: Perfect Compliments

Engel Curve for Cereal

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Inverse Demand

Summary
A commodity is said to satisfy the **law of demand** if the price increase leads to reduction in quantity demanded. Otherwise, the commodity is called a **Giffen good**.

- Examples of Giffen goods?
- Prada shoes: If they’re cheap and everyone’s wearing them, then ...
Price Offer Curve

The curve containing all the utility-maximizing bundles traced out as \( p_1 \) changes, with \( p_2 \) and \( m \) constant, is the \( p_1 \)-price offer curve.

Marshalian Demand Curve

The plot of the \( x_1 \)-coordinate of the \( p_1 \)-price offer curve against \( p_1 \) is the Marshalian demand curve for commodity 1.
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Price Offer Curve & Demand Curve

Fixed $p_2$ and $m.$

$p_1$ price offer curve

Downward-sloping demand curve

Good 1 satisfies LOD

$X_1^*$

Fixed $p_2$ and $m.$

$p_1$ price offer curve

Demand curve has a positively sloped part

Good 1 is Giffen

$X_1^*$
Perfect complements \((x_1, x_2)\) with \((p_1, p_2) = (1, 1)\)
Consider Coke and Pepsi.
Now consider cheesecakes and tea \((x_C, x_T)\):

**Cobb-Douglas Utility Function**

\[
\max_{x_C, x_T} u(x_C, x_T) = x_C x_T \quad \text{s.t.} \quad x_C + 2x_T = m.
\]

- MRS at \((x_C, x_T)\) is \(\frac{-x_C}{x_1}\).
Example 3: Cobb-Douglas

Marshalian Demand Curve for Cheesecakes

\[ \phi_C (m=20, p_C, p_T=2) = \frac{20}{p_C} \]

\[ \phi_C (m=10, p_C, p_T=2) = \frac{10}{p_C} \]
The initial question: how does the increase in the gas price affect Greg’s spending on community services ($\approx \text{"purchase" of volunteering work}$).

### Substitutes & Compliments

1. A cheesecake is a complement to tea if increase in the price of tea leads to decrease in cheesecake consumption.

2. A cheesecake is a substitute to tea if increase in the price of tea leads to increase in cheesecake consumption.

- Bottles & six-packs of Corona?
- Cereal & milk?
- Cobb-Douglas utility?

(cont’d from the previous example)

$$\max_{x_C, x_T} u(x_C, x_T) = x_C x_T \quad \text{s.t.} \quad x_C + 2x_T = m.$$  

$$\phi^T(m = 20, p_C, p_T = 2) = 10$$ is independent of the price of cheesecakes.
Inverse Demand Function

Inverse demand function $D(x_1)$ assigns the price at which the amount $x_1$ will be chosen, given $p_2$ and $m$. 
- So?
- Normalize $p_2 = 1$.
- Tangency condition:

$$p_1 = MRS(x_1^*, x_2^*).$$

$$\Rightarrow D(x_1^*) = p_1 = MRS(x_1^*, x_2^*) \quad (= \text{marginal willingness to pay})$$

- Inverse demand function denotes the marginal willingness to pay at each $x_1$ given $p_2$ and $m$.
- Note the inverse demand function shifts left and right if you change $p_2$ and $m$. 
Inverse Demand for Cheesecakes

\[ D(x_C) = \frac{20}{x_C} \text{ at } m=20 \]

\[ D(x_C) = \frac{10}{x_C} \text{ at } m=10 \]
- Characterize commodities by changing situational background.
- Income changes and associated graphs.
- Own-price changes and associated graphs.
- Cross-price changes and associated graphs.
- Meaning of the inverse demand function.