Third Exam

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Comments:

* You must clearly show your work in order to receive full credit on any problem.
* You are allowed to use a graphing calculator on this exam. You may NOT use a smartphone or any other device with internet capabilities.
* Turn in your equation sheet with your exam.
* If you need more space for calculations, a blank page is included at the end of the exam.
* You will lose points for numerical answers with missing or incorrect units.
* You will lose points for inappropriate use of sig-figs.
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**Problem 1 (20 points):** A bicycle wheel has an initial angular velocity of 1.10 rad/s.

a) If its angular acceleration is constant and equal to 0.275 rad/s$^2$, what is its angular velocity at time $t = 2.10$ s?

b) Through what angle has the wheel turned between time $t = 0$ s and $t = 2.10$ s?

For rotational motion with constant acceleration

$$\omega = \omega_0 + \alpha t = 1.10 \text{ rad/s} + \left(0.275 \text{ rad/s}^2\right)(2.10 \text{ s}) = 1.68 \text{ rad/s}$$

and (angular) displacement

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = (1.10 \text{ rad/s})(2.10 \text{ s}) + \frac{1}{2} \left(0.275 \text{ rad/s}^2\right)(2.10 \text{ s})^2 = 2.92 \text{ rad}.$$
Problem 2 (20 points): Small blocks, each mass $m$, are clamped at the ends and at the center of a rod of length $L$ and negligible mass.

a) Compute the moment of inertial of the system about an axis perpendicular to the rod and passing through the center of the rod.

b) Compute the moment of inertial of the system about an axis perpendicular to the rod and passing through a point one-forth of the length from one end.

The moment of inertia is given by

$$I = mr_1^2 + mr_2^2 + mr_3^2$$

For part a:

$$I = m \left( (\frac{L}{2})^2 + 0 + (\frac{L}{2})^2 \right) = \frac{1}{2} mL^2$$

and for part b:

$$I = m \left( (\frac{L}{4})^2 + (\frac{L}{4})^2 + (\frac{3L}{4})^2 \right) = \frac{11}{16} mL^2$$
**Problem 3 (20 points):** The flywheel of an engine has a moment of inertia $2.70 \text{ kg}\cdot\text{m}^2$ about its rotation axis.

a) What constant torque is required to bring it up to an angular speed of $430 \text{ rev/min}$ in a time of $8.50 \text{ s}$ starting from rest.

b) What is its final kinetic energy?

*Torque required to bring the flywheel up to speed can be determined from (rotational analog of impulse- momentum theorem)*

$$\tau \Delta t = I \omega$$

and so

$$\tau = \frac{I \omega}{\Delta t} = \frac{(2.70 \text{ kg}\cdot\text{m}^2) \cdot (430 \text{ rev/min}) \cdot (2\pi \text{ rad/rev}) \cdot (1 \text{ min}/60 \text{ s})}{8.50 \text{ s}} = 14.3 \text{ N}\cdot\text{m}$$

And the final kinetic energy is

$$I = \frac{1}{2} I \omega^2 = \frac{1}{2} (2.70 \text{ kg}\cdot\text{m}^2) \cdot (430 \text{ rev/min}) \cdot (2\pi \text{ rad/rev}) \cdot (1 \text{ min}/60 \text{ s})^2 = 2730 \text{ J}$$
**Problem 4 (20 points):** A diver comes off a board with arms straight up and legs straight down, giving her a moment of inertia about her rotation axis of $I_i = 18\text{ kg}\cdot\text{m}^2$. She then tucks into a small ball, decreasing this moment of inertia to $I_f = 3.6\text{ kg}\cdot\text{m}^2$. While tucked, she makes two complete revolutions in $t_1 = 1.1\text{ s}$. If she hadn’t tucked at all, how many revolutions would she have made in the $t_2 = 1.8\text{ s}$ from board to water?

*The energy is not always conserved, but momentum is and so we can use conservation of angular momentum*

\[ I_i\omega_i = I_f\omega_f \]

to express angular velocity before tucking

\[ \omega_i = \frac{I_f}{I_i}\omega_f. \]

With final (after tucking) angular velocity she made two complete revolutions and thus

\[ \omega_f t_1 = 4\pi \]
\[ \omega_f = \frac{4\pi}{t_1} \]

Therefore her initial (before tucking) angular velocity is

\[ \omega_i = \frac{I_f}{I_i}\left(\frac{4\pi}{t_1}\right) \]

with which in $t_2$ seconds she would have rotated by angle

\[ \theta = \omega_i t_2 = 4\pi \frac{I_f}{I_i}\frac{t_2}{t_1} \]

or would make

\[ \frac{\theta}{2\pi} = \frac{2 I_f t_2}{I_i t_1} = \frac{2 \cdot 3.6 \cdot 1.8}{18 \cdot 1.1} = 0.65 \text{ rev} \]

revolutions.
**Problem 5 (20 points):** A uniform ladder $X = 5.0 \text{ m}$ long rests against a frictionless, vertical wall with lower end $L = 3.0 \text{ m}$ from the wall. The ladder weights $W_l = 160 \text{ N}$. The coefficient of static friction between the foot of the ladder and the ground is $\mu_s = 0.40$. A man weighting $W_m = 740 \text{ N}$ climbs slowly up the ladder.

a) What is the maximum frictional force that the ground can exert on the ladder at its lower end?

b) What is the actual frictional force when the man has climbed 1.0 m along the ladder?

c) How far along the ladder can the man climb before the ladder starts to slip?

The angle of inclination is given by

$$\alpha = \arccos \left( \frac{5}{3} \right) = 0.93 \text{ rad}$$

This is a 2D equilibrium problem which gives us two translational conditions

$$\mu_s N_1 - N_2 = 0 \quad \text{(1)}$$

$$N_1 - W_l - W_m = 0 \quad \text{(2)}$$

(Where $N_1$ and $N_2$ are normal forces from the ground and from the wall respectively) and one more rotational conditions which we choose to be expressed for rotations around the foot of the ladder

$$N_1 \cdot 0 + \mu_s N_1 \cdot 0 - W_l \frac{L}{2} \cos \alpha - W_m z \cos \alpha + N_2 L \sin \alpha = 0$$

or

$$N_2 = \left( \frac{1}{2} W_l + \frac{z}{L} W_m \right) \frac{\cos \alpha}{\sin \alpha} \quad \text{(3)}$$

Using (2) the largest fictional force can be is

$$\mu_s N_1 = \mu_s (W_l + W_m) = 0.40 \cdot 900 \text{ N} = 360 \text{ N}$$

and from (1) and (3) for $z = 1.0 \text{ m}$ the frictional force is

$$\mu_s N_1 = N_2 = \left( \frac{1}{2} W_l + \frac{1}{5} W_m \right) \frac{\cos \alpha}{\sin \alpha} = 170 \text{ N}.$$ 

And so the ladder would start slipping when

$$\left( \frac{160 \text{ N}}{2} + \frac{z}{5.0 \text{ m}} 740 \text{ N} \right) 0.75 = 360 \text{ N}$$

or

$$z = 2.7 \text{ m}.$$
(Bonus) Problem 6 (20 points): Two round rods, one steel and the other copper, are joined end to end. Each rod is 0.750 m long and 1.50 cm in diameter. The combination is subject to a tensile force with magnitude 4000 N. (Young’s modulus for steel is $20 \times 10^{10}$ Pa and for copper is $11 \times 10^{10}$ Pa)

a) For steel rod, what is the strain?

Strain for the steel rod is given by

$$\frac{F}{YA} = \frac{4000 \text{ N}}{(20 \times 10^{10} \text{ Pa}) \left(3.14 \cdot (1.50 \times 10^{-2}/2)^2\right)} = 1.13 \times 10^{-4}$$

b) For steel rod, what is the elongation?

And thus elongation is

$$\Delta L = L \left(\frac{F}{YA}\right) = 8.49 \times 10^{-5} \text{ m}$$

c) For copper rod, what is the strain?

Strain for the copper rod is given by

$$\frac{F}{YA} = \frac{4000 \text{ N}}{(11 \times 10^{10} \text{ Pa}) \left(3.14 \cdot (1.50 \times 10^{-2}/2)^2\right)} = 2.06 \times 10^{-4}$$

d) For copper rod, what is the elongation?

And thus elongation is

$$\Delta L = L \left(\frac{F}{YA}\right) = 1.54 \times 10^{-4} \text{ m}$$