Econ 4935 Urban Economics

Lecture 1D: Hotelling’s Model

Instructor: Hiroki Watanabe

Fall 2012
1. Hotelling’s Model
2. Monopoly (Liz Alone)
3. Duopoly (Liz & Kenneth)
4. Oligopoly ($N \geq 2$)
5. Now We Know
### Hotelling’s Model
- Hot Dog Vendors in Rockefeller Plaza
- Vendor’s Decision
- Consumer’s Decision

2. Monopoly (Liz Alone)

3. Duopoly (Liz & Kenneth)

4. Oligopoly ($N \geq 2$)

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Hot Dog Vendors in Rockefeller Plaza

- **Firm’s location choice (Lecture 1B):**
  - Firm A’s location choice affects Firm B via bid rent.
  - Production level $\bar{y}$ (total size of office space) was given.
  - What if $\bar{y}$ is endogenous and firm’s location choice affects profit of other firms?
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### Hot Dog Vendors in Rockefeller Plaza

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Hot Dog Vendors in Rockefeller Plaza

Hampton at Gravois, St. Louis, MO
Hot Dog Vendors in Rockefeller Plaza

- Hotelling’s model\(^1\)
  - Consumers are spread evenly along a line segment \([0, 1]\) (Rockefeller Plaza).
  - Each consumer purchases one hot dog.

\(^1\)Cf Varian Chpt 25 [Var05].
Hot Dog Vendors in Rockefeller Plaza

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**Vendor’s Decision**

- There are 1, 2 or 3 vendors:
  1. Liz alone (monopoly)
  2. Liz and Kenneth (duopoly)
  3. Liz, Kenneth and Jack (oligopoly)

- Assume that the cost is sunk.
- Liz’s profit is \( 1 \cdot \text{sales volume} = \text{her market share} \).
- Same for Kenneth.
There are 1, 2 or 3 vendors:

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- Same for Kenneth.
Vendor’s location is given by $x^L$ and $x^K$.

Definition 1.1 (Mill & Delivered Price)

1. Mill price is the on-site price of a hot dog. Assume that hot dogs are $1 each.
2. Delivered price is the overall cost that a consumer pays for a hot dog, including ________. A consumer at $x$ ($0 \leq x \leq 1$) pays:

$$\text{Mill price} + \frac{|x - x^L|}{K}$$
Vendor’s location is given by $x^L$ and $x^K$.

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\[
\frac{1}{\text{mill price}} + \frac{\left|x - x^L \text{ or } K\right|}{\text{distance to the closest vendor}}.
\]
Consumer’s Decision

- Mill Price: $\text{Mill}(x) = 1$
- Delivered Price: $\text{Delivered}(x) = 1 + |x - 0|$

Graph showing the relationship between location (miles) and mill price and delivered price ($). The delivered price is a linear function of location, increasing with distance from the mill.
1 Hotelling’s Model

2 Monopoly (Liz Alone)
   - What Is Best for Liz
   - What Is Best for Everyone

3 Duopoly (Liz & Kenneth)

4 Oligopoly \( N \geq 2 \)

5 Now We Know
What Is Best for Liz

Mill Price and Delivered Price ($)

Location (miles)

Mill(x)=1
Delivered(x)=1+|x-0|

Model

Monopoly
Duopoly
Oligopoly
What Is Best for Liz

Mill Price and Delivered Price ($)

Location (miles)

Mill(x) = 1
Delivered(x) = 1 + |x - 0.25|

Watanabe Econ 4935 ID Hotelling’s Model
What Is Best for Liz

**Mill Price and Delivered Price ($)**

- **Mill(x) = 1**
- **Delivered(x) = 1 + |x - 0.5|**
What Is Best for Everyone

\[ x^L \quad \text{Liz’s profit} \]

Liz’s optimum

Social optimum
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What Is Best for Everyone

\[ x^L \] Liz’s profit

Liz’s optimum

Social optimum
Hotelling’s Model

Monopoly (Liz Alone)

Duopoly (Liz & Kenneth)
- What Is Best for Everyone
- What Is Best for Liz & Kenneth
- Nash Equilibrium vs Social Optimum
- Example

Oligopoly ($N \geq 2$)

Now We Know
Fact: the **social optimal** outcome is ...
What Is Best for Everyone

Mill Price and Delivered Price ($)

\[ \text{Mill}(x) = 1 \]
\[ \text{Delivered}^L(x) = 1 + |x - 0.25| \]
\[ \text{Delivered}^K(x) = 1 + |x - 0.75| \]
Exercise 3.1 (Duopolistic Competition)

Starting from \((x^L, x^K) = (0, 1)\), take turns switching your locations to maximize your profit.

- In case of \(x^L = x^K\), split the market equally in half.
- No threatening or cartels.

1. Where will you end up and how much will you earn?
2. Do we get \((x^L, x^K) = (.25, .75)\) or \((.75, .25)\) in the end?
What Is Best for Liz & Kenneth

- Mill Price: $1$
- Delivered Price to Liz: $1 + |x - 0|$
- Delivered Price to Kenneth: $1 + |x - 1|$

Location (miles) vs. Mill Price and Delivered Price ($\text{\$})$
What Is Best for Liz & Kenneth

\[ \text{Mill}(x) = 1 \]
\[ \text{Delivered}^L(x) = 1 + |x - 0| \]
\[ \text{Delivered}^K(x) = 1 + |x - 0.05| \]
What Is Best for Liz & Kenneth

\[ \text{Mill Price and Delivered Price ($)} \]

\[ \text{Mill}(x) = 1 \]

\[ \text{Delivered}_L(x) = 1 + |x - 0.1| \]

\[ \text{Delivered}_K(x) = 1 + |x - 0.05| \]
What Is Best for Liz & Kenneth

\[
\begin{align*}
\text{Mill Price and Delivered Price ($)} \\
\text{Mill}(x) &= 1 \\
\text{Delivered}_L(x) &= 1 + |x - 0.5| \\
\text{Delivered}_K(x) &= 1 + |x - 0.5|
\end{align*}
\]
What Is Best for Liz & Kenneth

![Graph showing Mill(x)=1, Delivered_L(x)=1+|x-0.5|, and Delivered_K(x)=1+|x-0.55|, with location (miles) on the x-axis and Mill Price and Delivered Price ($) on the y-axis.]

- Mill(x) = 1
- Delivered_L(x) = 1 + |x - 0.5|
- Delivered_K(x) = 1 + |x - 0.55|
What Is Best for Liz & Kenneth

The end result is called a Nash equilibrium.
Definition 3.2 (Nash Equilibrium)

is the location \((x^{LNE}, x^{KNE})\) such that none of the vendor can profit by **unilaterally** changing its location.

- For duopoly, the Nash equilibrium \((x^{LNE}, x^{KNE}) = (0.5, 0.5)\).
Discussion 3.3 (Social Optimal Choice)

1. How much do Kenneth and Liz earn with Nash equilibrium (.5, .5) and with social optimal outcome (.25, .75)?

2. Can they reach (.25, .75) on their own? If not, why not?
Discussion 3.3 (Social Optimal Choice)

1. How much do Kenneth and Liz earn with Nash equilibrium (.5, .5) and with social optimal outcome (.25, .75)?

2. Can they reach (.25, .75) on their own? If not, why not?
Nash Equilibrium vs Social Optimum

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<tbody>
<tr>
<td>Social Optimum</td>
<td>(.25, .75)</td>
<td>1.125 ☹</td>
</tr>
<tr>
<td>Social Optimum</td>
<td>(.75, .25)</td>
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</tr>
<tr>
<td>Nash Equilibrium</td>
<td>(.5, .5)</td>
<td>1.250 😞</td>
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Deadweight loss of $\text{______}$. 

Compare:
Nash Equilibrium vs Social Optimum

- **Compare:**

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<td>1.250 😞</td>
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- **Deadweight loss** of $____.
Nash Equilibrium vs Social Optimum

- Why can’t Liz and Kenneth choose (.25, .75) on their own?
Nash Equilibrium vs Social Optimum

Mill Price and Delivered Price ($)

- Mill(x) = 1
- Delivered^L(x) = 1 + |x - .25|
- Delivered^K(x) = 1 + |x - .75|

Location (miles)
Nash Equilibrium vs Social Optimum

Mill Price and Delivered Price ($)
- Mill(x) = 1
- Delivered^L(x) = 1 + |x - 0.25|
- Delivered^K(x) = 1 + |x - 0.3|

Location (miles)
Nash Equilibrium vs Social Optimum

\[ \text{Mill Price and Delivered Price (\$)} \]

\[ \text{Mill}(x) = 1 \]

\[ \text{Delivered}^L(x) = 1 + |x - 0.5| \]

\[ \text{Delivered}^K(x) = 1 + |x - 0.5| \]
Example 3.4 (Rockefeller Square)

Rockefeller Square is a 10-by-10 mile square and there are 100 consumers uniformly distributed over the Square. Location in the Square is identified by \((x, y)\), where \(x (0 \leq x \leq 10)\) measures longitude and \(y (0 \leq y \leq 10)\) measures latitude. There are two hot-dog vendors, Liz and Kenneth. The mill price is $1 for both. Production cost is sunk and already off their plates.

1. Suppose that \((x_L, y_L; x_K, y_K) = (0, 0; 10, 10)\). Show their market shares on a map.

2. Suppose that \((x_L, y_L; x_K, y_K) = (3, 1; 1, 5)\). Show their market shares on a map.

3. Find the Nash equilibrium.

4. Show that \((x_L, y_L; x_K, y_K) = (0, 0; 10, 10)\) yields the same profit as in 3 but it is not sustainable.
Example
Example

- Liz’s Share
- Kenneth’s Share
1. Hotelling’s Model

2. Monopoly (Liz Alone)

3. Duopoly (Liz & Kenneth)

4. Oligopoly ($N \geq 2$)
   - $N = 3$
   - $N \geq 4$

5. Now We Know
3 vendors: Liz, Kenneth and Jack Donaghy.

Fact: socially efficient location is 
\((x^L, x^K, x^D) = (1/6, 3/6, 5/6)\)^2.

^2 not necessarily in this order.
3 vendors: Liz, Kenneth and Jack Donaghy.

Fact: socially efficient location is
\((x^L, x^K, x^D) = (1/6, 3/6, 5/6)^2\).

\(^2\)not necessarily in this order.
$N = 3$

\[ \text{Location (miles)} \]

\[ \text{Mill Price and Delivered Price (\$)} \]

- **Mill Price**:
  - Mill(x) = 1

- **Delivered Prices**:
  - Delivered$^L(x) = |x - 1/6|
  - Delivered$^K(x) = |x - 3/6|
  - Delivered$^D(x) = |x - 5/6|

---

**Graphical Representation**

- Yellow line: Mill(x) = 1
- Red line: Delivered$^L(x) = |x - 1/6|
- Blue line: Delivered$^K(x) = |x - 3/6|
- Green line: Delivered$^D(x) = |x - 5/6|

**Axes**
- Y-axis: Mill Price and Delivered Price (\$)
- X-axis: Location (miles)

**Range**
- Location (miles): 0 to 1
- Price: 0 to 1
Exercise 4.1 (Oligopolistic Competition)

Starting from \((x^L, x^K, x^D) = (0, 0, 0)\), take turns switching your locations to maximize your profit.

1. Where will you end up and how much will you earn?
2. Is \((x^L, x^K, x^D) = (1/6, 3/6, 5/6)\) sustainable?
3 vendors: Liz, Kenneth and Jack.

Consider:

1. All the vendors locate at different places.
2. \( L = K = D \).
3. \( L \neq K \neq D \).
3 vendors: Liz, Kenneth and Jack.

Consider

1. All the vendors locate at different place.
2. $x^L = x^K = x^D$.
3. $x^L = x^K \neq x^D$. 
3 vendors: Liz, Kenneth and Jack.

Consider

1. All the vendors locate at different place.
2. \( x_L = x_K = x_D \).
3. \( x_L = x_K \neq x_D \).
3 vendors: Liz, Kenneth and Jack.

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1. All the vendors locate at different place.
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Consider

1. All the vendors locate at different place.
2. $x^L = x^K = x^D$.
3. $x^L = x^K \neq x^D$. 
\[ N = 3 \]

The graph illustrates the Mill Price and Delivered Price ($ \text{Mill}(x) = 1$, \text{Delivered}^L(x) = |x - .1|, \text{Delivered}^K(x) = |x - .3|, \text{Delivered}^D(x) = |x - .9|) over the location (miles) from 0 to 1.
$N = 3$

- **Model**
  - Monopoly
  - Duopoly
  - Oligopoly

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<td></td>
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<td>Delivered$^K(x)=</td>
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<tr>
<td></td>
<td>Delivered$^D(x)=</td>
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- **Graph**
  - Mill Price and Delivered Price ($) vs Location (miles)
  - Mill(x) = 1
  - Delivered$^L(x)=|x−.2|
  - Delivered$^K(x)=|x−.3|
  - Delivered$^D(x)=|x−.8|

- **Equation**
  - Mill Price and Delivered Price ($)
  - Location (miles)
$N = 3$

- **Mill Price**: $\text{Mill}(x) = 1$
- **Delivered Price**:
  - $\text{Delivered}^L(x) = |x - 0.2|$
  - $\text{Delivered}^K(x) = |x - 0.2|$
  - $\text{Delivered}^D(x) = |x - 0.2|$
\[ N = 3 \]

![Graph](image-url)

- **Mill Price**:
  - \([\text{Mill}(x) = 1] \)

- **Delivered Price**:
  - \([\text{Delivered}^L(x) = |x - 0.2|] \)
  - \([\text{Delivered}^K(x) = |x - 0.2|] \)
  - \([\text{Delivered}^D(x) = |x - 0.8|] \)
$N = 3$

- Mill Price and Delivered Price ($$)

- Location (miles)

- Mill(x) = \(|x - 0.2|\)
- Delivered_L(x) = |x - 0.3|
- Delivered_D(x) = |x - 0.8|
  
- Model
  - Monopoly
  - Duopoly
  - Oligopoly
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For $N = 3$, there is no Nash eqm.
There are Nash equilibria for $N \geq 4$. 
1. Hotelling’s Model
2. Monopoly (Liz Alone)
3. Duopoly (Liz & Kenneth)
4. Oligopoly ($N \geq 2$)
5. Now We Know
- Hotelling’s model
- Social optimality vs individual optimality.
- Nash equilibrium
Map du Jour

Source: http://www.worldmapper.org/
Today’s color theme is provided by courtesy of Thai Airways.
deadweight loss, 28
delivered price, 9
duopoly, 17
mill price, 9

monopoly, 12
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