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Much literature has addressed the size distribution of cities:

- Zipf’s Law (Rank-size Rule)
- Gibrat’s Law

Examples:
- Gabaix (1999a,b)
- Rossi-Hansberg and Wright (2007)
- Duranton (2006, 2007)
Zipf’s Law: The rank of a city multiplied by its population is constant.

1. Population of NY = c.
   ...

Empirically fits the data in many countries and many time periods.

Deviations at top and bottom of distribution well documented and discussed (e.g. Gabaix and Ioannides, 2004).

\[ \text{rank} \cdot \text{size} = c \Rightarrow \log(\text{rank}) + \log(\text{size}) = \log c. \]
The city-size distribution has a fat tail.
Gaussian distribution cannot explain C-S dist.
20-80 rule: upper 20% explains 80% of the total.

20-86
2-8.7 (top 5)
.4-3.8 (NYC alone)

Figure:

Standard methodology
- plausible assumptions
- structural form
- reduced form
- fit the data

Models of city-size distribution
- unconventional assumptions
- structural form
- reduced form
- robust fact (Zipf’s law)
Unconventional assumption: no insurance...
even though the models of Zipf’s law provide a perfect setting for insurance:

1. In each period, all agents observe the state of the world.
2. Then they make their decisions.
3. No information asymmetry comes into play.

Eeckhout (2004)’s model

Equilibrium without insurance

For any \( i \) (city) and \( t \) (time),

1. Utility maximization.
   - Consumers observe technological shock \( A_{i,t} \) before they make their decision in time \( t \).

2. Profit maximization.

3. Commodity market clearance.
   - Output \( Y_{i,t} = A_{i,t}a_+(S_{i,t})L_{i,t} \).
   - \( a_+(S_{i,t}) \) = positive externality, \( S_{i,t} \) = population, and \( L_{i,t} \) = labor supply.

4. Housing market clearance.

5. Labor market clearance.
Eeckhout (2004)’s model

**Equilibrium distribution without insurance**

\[ v(S_{i,t}, A_{i,t}) = \bar{v} \]  for \( i = 1, \ldots, l \). Constant utility across cities in each time period.

Law of motion for \( A_{i,t} \):

\[ A_{i,t} = (1 + \sigma_{i,t})A_{i,t-1} \quad (\sigma_{i,t} \sim i.i.d., E(\sigma_{i,t}) = 0.) \]

implies

\[ \log S_{i,T} = \log S_{i,0} + \sum_{t=0}^{T} \epsilon_{i,t} \sim \text{Normal distribution} \]

and thus we obtain Gibrat’s law: Proportional stochastic growth \( \Rightarrow \) lognormal distribution.

Consumers don’t want to move to highest productivity city (or all want to move to one place) due to a negative congestion externality on the consumer side of the model.
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Four Means of Hedging against Risk

Agents can smooth the consumption stream by:
1. relocating
2. self-insurance
3. insurance markets
4. futures markets

- Literature assumes 1.
- We use all the above.
Four Means of Hedging against Risk

1/4. Relocation

![Graph showing income over time with locations NY and LA, with points A and B indicating risk]

2/4. Self-insurance

![Graph showing income over time with locations NY and LA, with risk points labeled]
3/4. Insurance Markets

- Consumers make a contract to sell labor and in return receive fixed consumption commodity and housing regardless of the realized state of nature.
Adverse selection
- Everyone knows the state.

Local gov’t might fake the state to extract more money from the federal gov’t.
- There is no local gov’t in the model.
- The state is known to everyone before they make location decisions.

Time inconsistency
- We assume the commitment on the part of consumers:
  - One-period commitment for insurance markets.
  - Longer periods of commitment for self insurance.

Costs of insurance contracts
- Maybe.

The models of Zipf’s law provide a perfect setting for insurance and...
Four means of hedging against risk

Agents can smooth the consumption stream by:
1. relocating
2. self-insurance
3. insurance markets
4. futures markets

- Literature: 1 leads to lognormal distribution.
- Our model:
  - 1 becomes irrelevant.
  - 2, 3 and 4 lead to uniform distribution.
  - Insurance is a substitute for relocation.
Eeckhout (2004)’s model with insurance

- Start with uniform distribution of consumers.
- Insure using saving and borrowing or insurance pools (or both).
- Utility is the same period by period as the utility found in Eeckhout with movement.

<table>
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<tr>
<th>non-moving cost / non-zero moving cost / insurance</th>
<th>no residual uncertainty</th>
<th>residual uncertainty</th>
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<tbody>
<tr>
<td>unavailable</td>
<td>move (literature)</td>
<td>move</td>
</tr>
<tr>
<td>available</td>
<td>move or stay</td>
<td>stay</td>
</tr>
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- We conjecture that other models can be modified similarly. (We use Eeckhout as an example).

- So, consumers stay put by hedging the risk with insurance instead of relocation. We need a more drastic shock to observe consumer movement.
- What kind?
- In each industry, only the city with the best technology survives.
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- City \((j = 1, \cdots, m)\), commodity \((i = 1, \cdots, n)\).
- Technology: \(y_{ijt} = A_{ij}l_{ijt}\)
- No city can host multiple industries at one time.
- A "city" is defined by a set of firms who experience a common technological shock.
Consumer solves
\[
\max u(t) = \sum_{i=1}^{n} \frac{1}{n} c_i(t)^{\gamma}
\]
subject to \[\sum_{i=1}^{n} p_i(t)c_i(t) = w(t).\]

Profit maximization implies
For \(i = 1, \ldots, n\), for \(j^*\) with \(A_{ij^*t} = \max_{1 \leq j \leq m, 0 \leq t' \leq t} A_{ijt'}\),
\[p_i(t)A_{ij^*t} = w(t).\]

All in all,
\[l_{ij^*t} = k(t)(A_{ij^*t})^{\frac{\gamma}{1-\gamma}} \quad \Rightarrow \quad A_{ij^*t} = \left(\frac{l_{ij^*t}}{k(t)}\right)^{\frac{1-\gamma}{\gamma}}. \quad (1)\]

Fisher-Tippett Theorem
Let \(\{x_i\}\) denote a sequence of random variables. Maximum value \(x^* := \max \{x_i\}\) has the distribution function of the form
\[F_{GEV}(x^*) = \begin{cases} 
\exp\left\{-\left[1 + \frac{\xi b(x^* - u) - \frac{1}{\xi}\right]\right\} & \text{when } \xi \neq 0 \\
\exp\{-\exp[-b(x^* - u)]\} & \text{when } \xi = 0
\end{cases}\]

The sequences of random variables do not have to be i.i.d.

- Function (2) is called generalized extreme value distribution.
- Original versions of F-T Theorem require i.i.d.
- Modern versions of F-T Theorem do not.
- Like law of large numbers for maximum of random variables rather than average.
- $\xi = 0 \Rightarrow$ Gumbel
- $\xi < 0 \Rightarrow$ Reverse Weibull
- $\xi > 0 \Rightarrow$ Fréchet

In our model, for $F(l) = 1 - \frac{\text{rank}}{m}$, substituting $x^*$

with $A_{ij^*} = \left( \frac{I_{ij^*}t}{\kappa(t)} \right)^{\frac{1-\gamma}{\gamma}}$, 

$$\log(F(l)) = \begin{cases} 
- \left\{ 1 + \xi b \left[ \left( \frac{l}{\kappa(t)} \right)^{\frac{1-\gamma}{\gamma}} - u \right] \right\}^{-\frac{1}{\xi}} & \text{when } \xi \neq 0 \\
- \exp \left\{ -b \left[ \left( \frac{l}{\kappa(t)} \right)^{\frac{1-\gamma}{\gamma}} - u \right] \right\} & \text{when } \xi = 0
\end{cases}$$
Consumers would rather move to a city with production than insure. Since only a few cities produce, insurance is very expensive.

Consumer may save to smooth $w(t)$ but spatial distribution is unchanged.

Multiple cities can produce the same commodity.
- More than one city survives if transportation cost is positive.
- Fisher-Tippett theorem applies to upper order statistics as well.

We assumed $A_{ijt}$ is i.i.d. across $i$ and $t$.
- Technological development is path dependent.
- Cities located near the current city in production are more likely to innovate.

Modern version of Fisher-Tippett theorem allows the above scenarios.
- We just need stationarity and asymptotic block independency.
Introduction

Risk Hedging

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Empirical Implementation

Our result (GEV)

Empirical and estimated CDF

Estimation with Frechet

Empirical CDF

Estimated CDF

Residual

Population in log scale

Empirical and estimated CDF

Estimation with lognormal

Empirical CDF

Estimated CDF

Residual

Population in log scale

Eeckhout’s result (Lognormal)
- Estimates maximize likelihood function (MLE).
- Kolmogorov-Smirnov statistic $: \sup \{ \text{Empirical CDF}(l) - \text{Estimated CDF}(l) \}$.

<table>
<thead>
<tr>
<th></th>
<th>Lognormal</th>
<th>GEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Eeckhout’s)</td>
<td>0.018913</td>
<td>0.006967</td>
</tr>
<tr>
<td>KS statistics</td>
<td></td>
<td></td>
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1. Eeckhout’s model focuses on 2nd-best allocations.
   - Efficiency loss due to externality (both positive and negative).
2. Duranton’s model focuses on 2nd-best allocations.
   - Efficiency loss due to imperfect competition.
3. Our model focuses on 1st-best allocations.

**Conclusion**

1. It is natural to use self-insurance or insurance.
2. If we allow insurance to smooth out the consumption stream, consumers stay put whether they face city-level or aggregate risks.
3. Resulting distribution is uniform and robust against additional assumptions (moving cost or residual uncertainty).
4. Extreme shocks (kicks in behinds) are needed to observe consumer migration.
5. Extreme value theory is employed.
6. Compelling empirical result.
Extension

1. Does the model explain other phenomena (rent, wages, prices) in a consistent manner? In particular, if land is added to the model, although equilibrium utility will be equal across location, both wage and land rent can differ across locations.

2. Empirical investigation of substitutability between insurance and migration.

3. Transportation cost: merging our model with the trade model of Eaton and Kortum (2002) — no labor/consumer mobility in that model.