Econ 4601 Urban & Regional Economics

Lecture 5: Utility Maximization Problems

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- So far we have discussed:
  - \$(Lecture 2)
  - $\succeq$ (Lecture 3)
  - ☺ (Lecture 4)
- Now combine all the above to predict Greg's consumption behavior given his budget constraint and preferences.
- Situational background:
  - Greg wants to have as many cheesecakes and tea as possible.
  - Greg's budget constraint does not allow him to choose (x_C, x_T) =(infinite, infinite).
  - Greg has to make choices.

- We assume that Greg picks the most preferred combination among what he can afford.
- The framework to analyze Greg's choice behavior is called utility maximization problem (UMP for short).

- Greg's decision making process is summarized as follows:

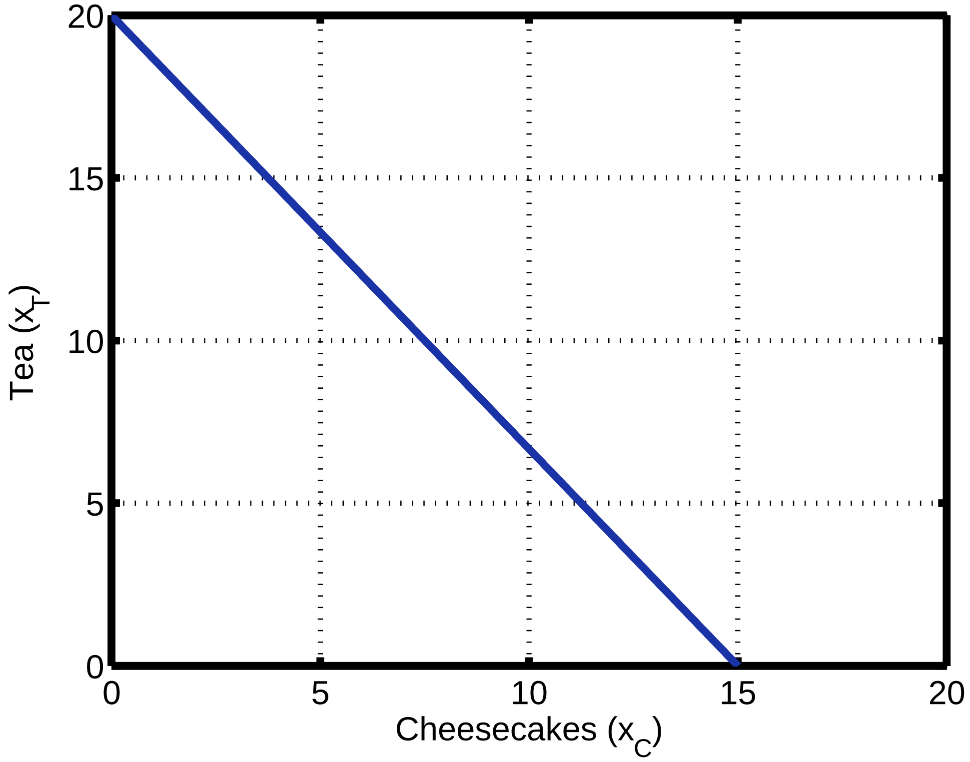
Utility Maximization Problem

Greg chooses the bundle (x_C, x_T) that gives him the highest utility level ☺ among the affordable bundles. In other words, he

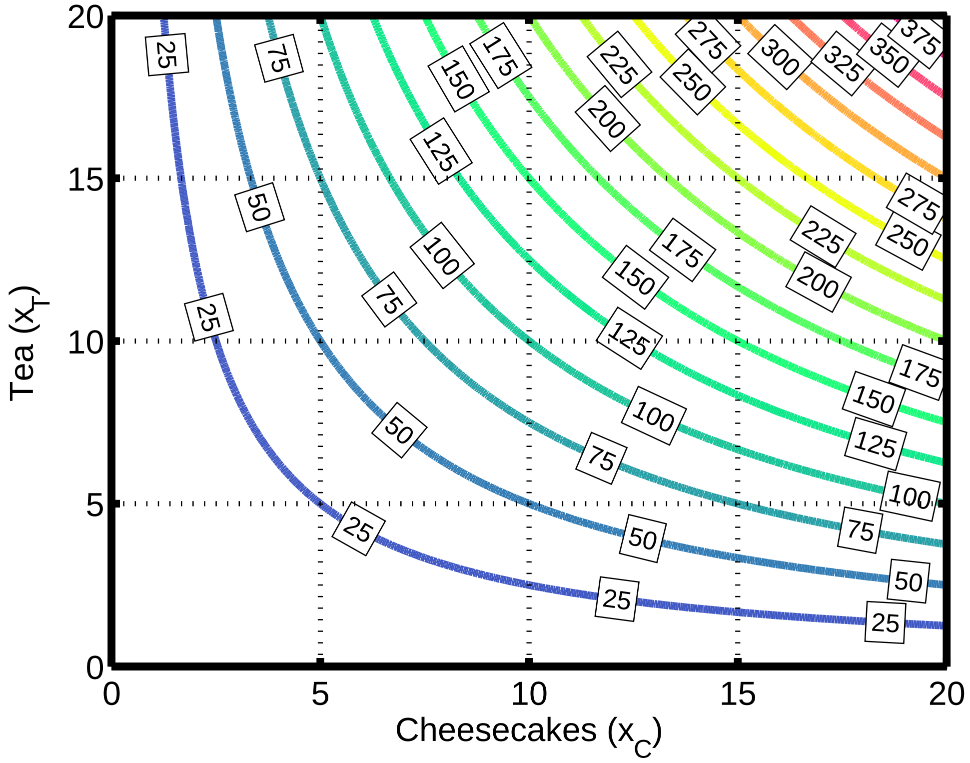
$$\max u(x_C, x_T) \quad \text{subject to} \quad p_C x_C + p_T x_T = m.$$

- How exactly do we find Greg's optimal bundle $x^* = (x_C^*, x_T^*)$?
- Consider the case when
  - $u(x_C, x_T) = x_C x_T$
  - $m = 60$,
  - $(p_C, p_T) = (4, 3)$.

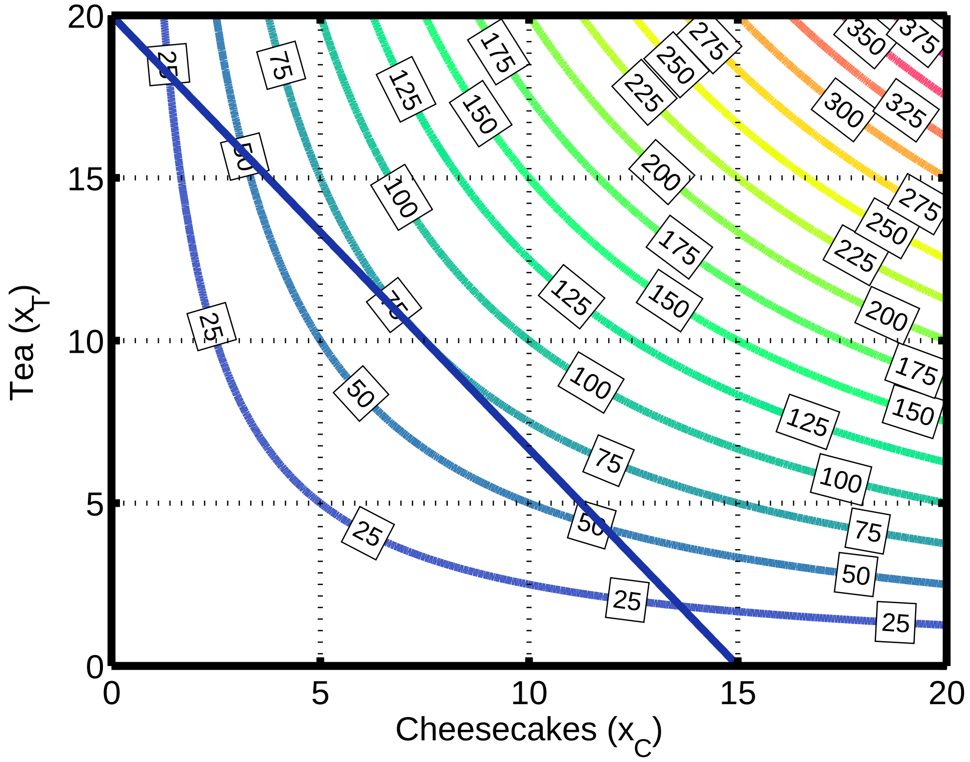
Budget Line Meets Indifference Curves



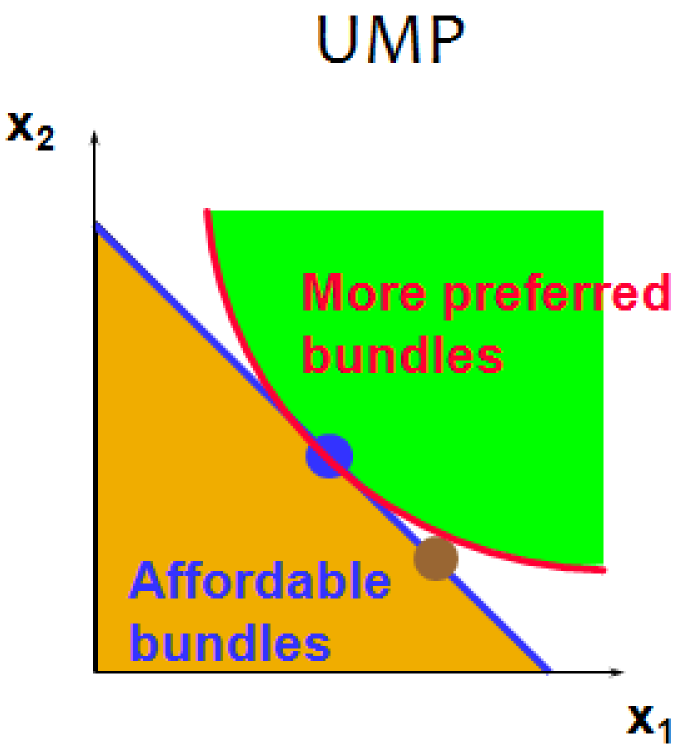
Budget Line Meets Indifference Curves



Budget Line Meets Indifference Curves



Budget Line Meets Indifference Curves



Tangency

Tangency Condition

At the optimal bundle, the indifference curve is tangent to the budget constraint (for standard preferences), i.e., they both have the same slope at x^*.

- What does the tangency condition imply?
- The slope of IC denotes MRS (MWTP): the cups of tea Greg is willing to give up for one slice of cheesecake.
- The slope of budgte line denotes relative price (op. cost): the cups of tea Greg has to give up for one slice of cheesecake.
- Greg's idea of the tea's worth coincides with market's idea of the tea's worth when he chooses the right amount.

- What if the tangency condition is not met?
- Suppose MRS is larger than the relative price (IC is steeper than the budget line).
  - Greg is willing to give up 30 cups of tea for a slice of cheesecake,
  - while he has to sell only $\frac{4}{3}$ cups of tea to buy a slice of cheesecake.
  - It's wise to increase cheesecake consumption and reduce cups of tea.
  - The current bundle is not optimal.

- Suppose MRS is smaller than the relative price (IC is steeper than the budget line).
  - Greg is willing to give up .1 cups of tea for a slice of cheesecake,
  - while he has to sell $\frac{4}{3}$ cups of tea to buy a slice of cheesecake.
  - It's wise to reduce cheesecake consumption and increase cups of tea.
  - The current bundle is not optimal.

- Where exactly is the bundget line tangent to the indifferent curve?
- We need a little bit of calculus to find Greg's optimal bundle.

UMP

$\max u(x_C, x_T) = x_C x_T \quad$ subject to $\quad 4x_C + 3x_T = 60.$

- MRS at (x_C, x_T) is given by $\frac{-x_T}{x_C}$.
- The relative price is $\frac{-4}{3}$.

- Tangency condition:

$$\frac{-x_T}{x_C} = \frac{-4}{3} \Rightarrow x_T = \frac{4}{3}x_C. \quad (1)$$

- There are lots of bundles (x_C, x_T) satisfying (1).
- The bundle also has to be affordable:

$$4x_C + 3x_T = 60 \Rightarrow 4x_C + 3\frac{4}{3}x_C = 60.$$

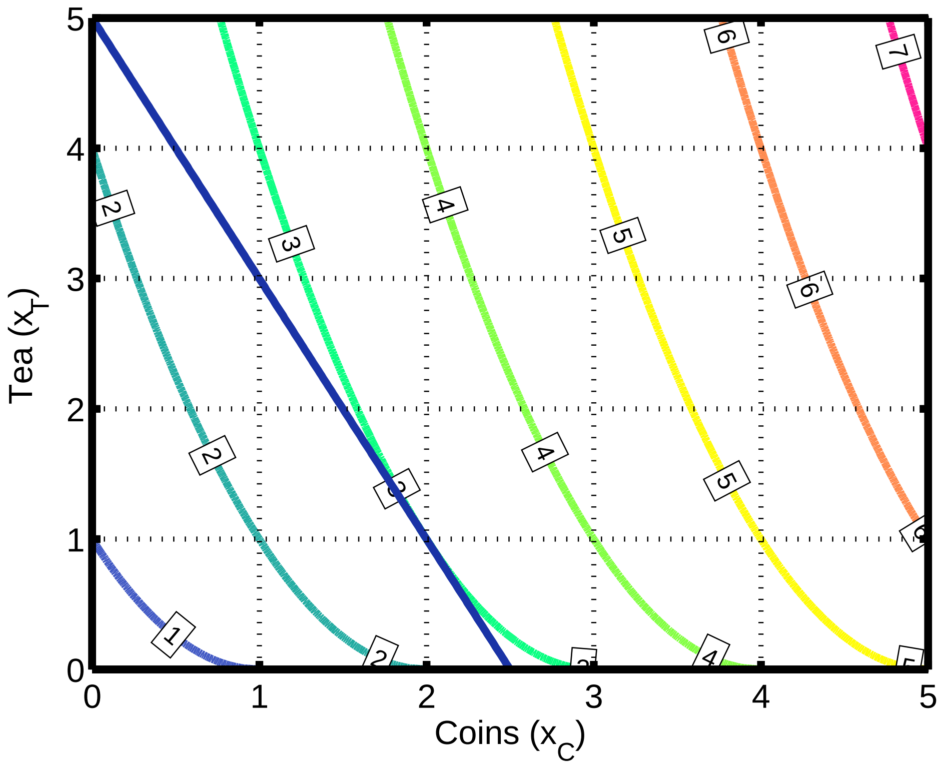
- $x_T = 7.5$ and $x_C = 10$.
- Conclude $x^* = (10, 7.5)$.

Exercise

Greg spends his income (\$5) on coins (x_C) and tea (x_T). Price is given by $(p_C, p_T) = (2, 1)$. His preferences are represented by $u(x_C, x_T) = x_C + \sqrt{x_T}$.

1. Write down the budget line.
2. Write Greg's utility maximization problem.
3. What is the tangency condition in this example? (MRS at (x_C, x_T) is $-2\sqrt{x_T}$).
4. Which bundle will Greg buy?

Find the Exact Solutions

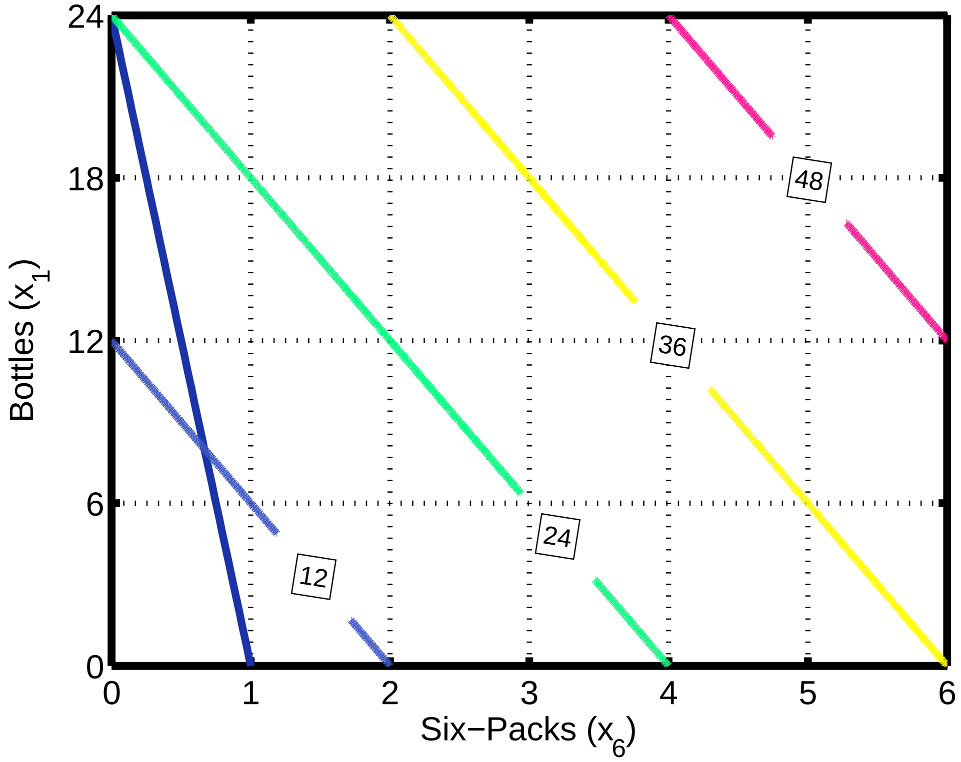


- There are some exceptions where tangency condition does not apply.

Perfect Substitutes

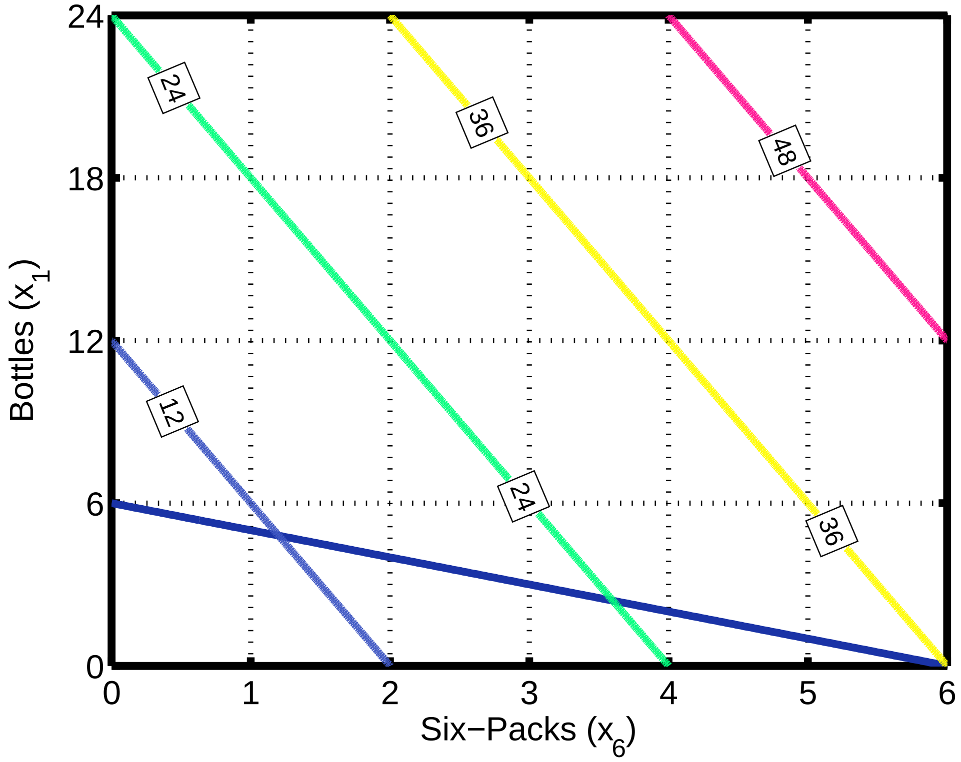
- Consider six-packs and bottles of Corona (x_6, x_1).
- Greg's MRS is 6 everywhere.
- Consider three cases:
  1. Market rate of exchange is higher than 6.
  2. Market rate of exchange is lower than 6.
  3. Market rate of exchange is exactly 6.

1. Market rate of exchange is higher than 6.
   - Let's say the market rate of exchange is 24 (i.e., 24 bottles of Corona is traded for 1 six-pack).
   - Greg is willing to give up 6 bottles for 1 six-pack.
   - Greg will spend all his income on six-packs.
   - This type of solution is called a corner solution (e.g., $(8, 0)$, $(35, 0)$, $(0, 68)$ etc.)

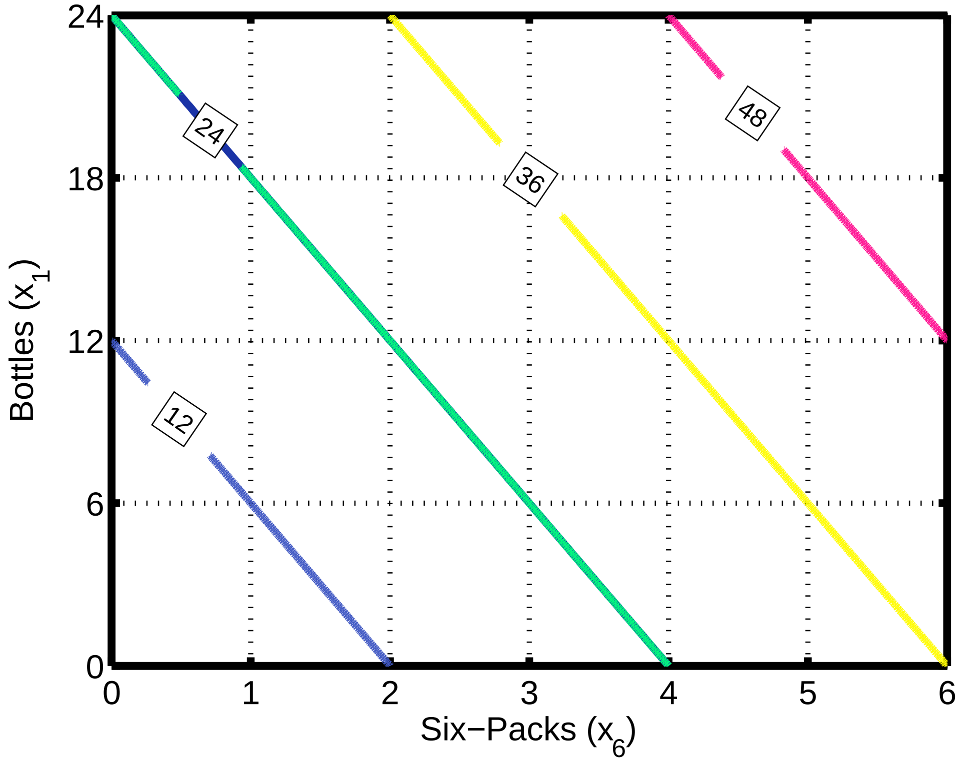


2. Market rate of exchange is lower than 6.
   - Let's say the market rate of exchange is 1 (i.e., 1 bottle of Corona is traded for 1 six-pack).
   - Greg is willing to give up 6 bottles for 1 six-pack.
   - Greg will spend all his income on bottles.

Intro Solving UMP Extreme Cases Σ
Perfect Substitutes



3. Market rate of exchange is exactly 6.
   - The market rate of exchange is 6 (i.e., 6 bottles of Corona is traded for 1 six-pack).
   - Greg is willing to give up 6 bottles for 1 six-pack.
   - Greg can choose any bundle on his budget line.



Discussion

- If a bottle of Corona is sold at \$2, a six-pack is usually sold at less than \$12.
- Does that mean nobody buys Corona by the bottle?

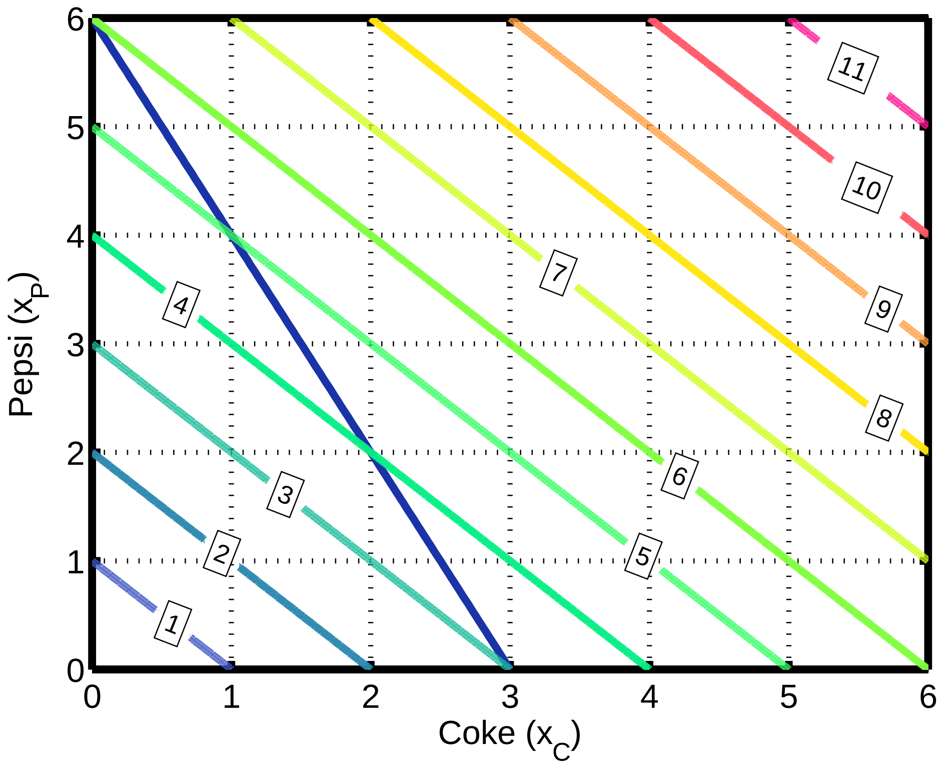
- Dharma's MRS might be less than 6.
- She is willing to give up 1 six-pack for 4 bottles (MRS$=4$).
- So, there are Corona sold by the bottle, and there are people who buy Corona by the bottle.

Exercise

Greg has \$12 and spend his income on Coke & Pepsi (x_C, x_P). Suppose Coke is sold at \$4 while Pepsi is sold at \$2.

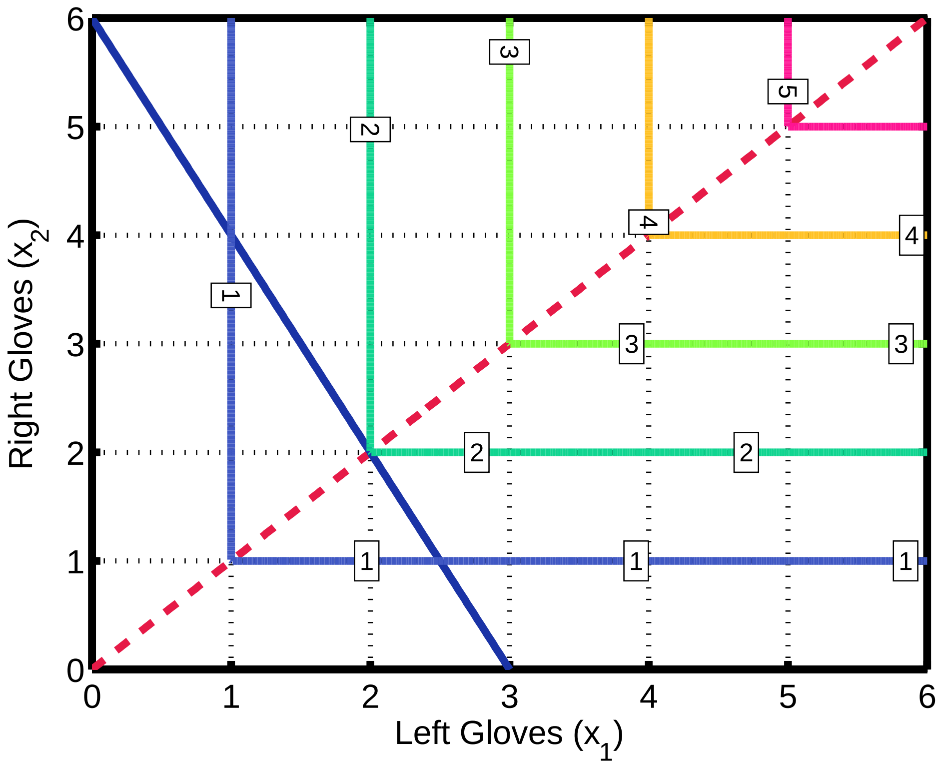
1. Find Greg's optimal bundle.
2. Confirm your answer using indifference curves and the budget line.

Perfect Substitutes



Perfect Complements

- For perfect complements, MRS is not defined.
- That does not mean the solution to UMP does not exist.
- Consider left gloves (x_1) and right gloves (x_2).



- Optimal bundle x^* for left gloves and right gloves is

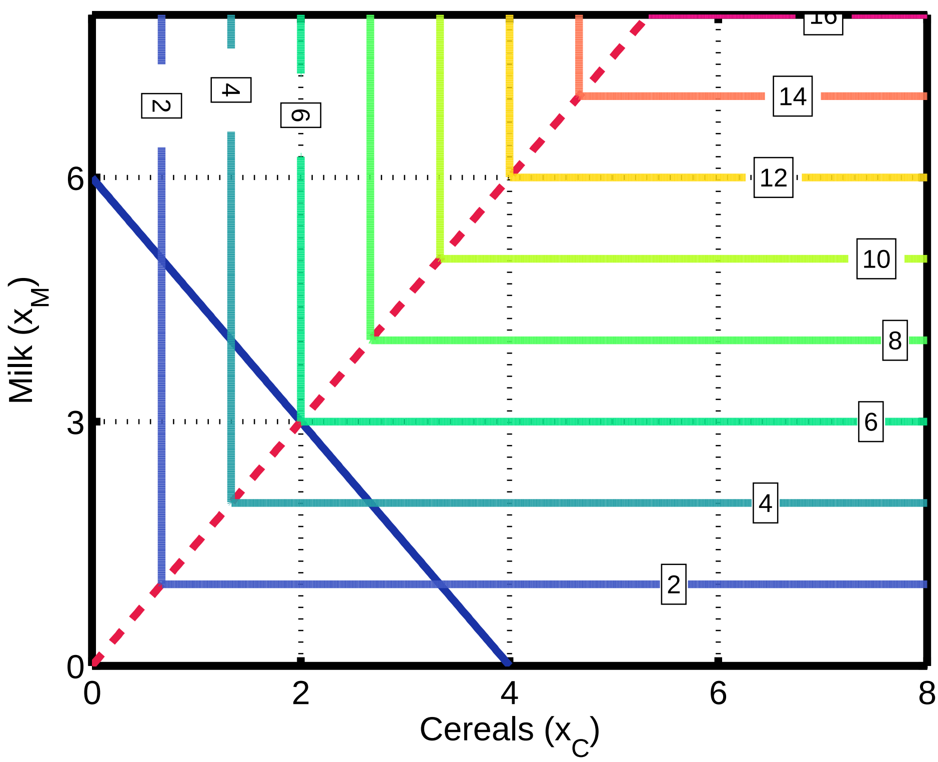
  1. on the budget line and
  2. on the 45 degree line.

Exercise

Consider a bundle (cereal, milk)$=(x_C, x_M)$.

- Greg says he can't have cereals without milk and the only time he drink milk is when he eats his cereals.
- Greg's preferred cereal-milk ratio is 2 to 3.
- $(p_C, p_M) = (6, 4)$.
- $m = 24$.

1. Draw his budget line.
2. Draw indifference curves at $(2, 3)$ and $(4, 6)$.
3. Mark the bundle Greg will choose on your graph.



- Greg's preferred bundles are lined up on the ray $x_M = \frac{3}{2}x_C$.
- The optimal bundle is
  1. on the ray $x_M = \frac{3}{2}x_C$ and
  2. on the budget line $x_M = \frac{-3}{2}x_C + 6$.

- How to set up the utility maximization problem.
- Tangency condition for standard cases.
- Optimal bundles for extreme preferences.