Econ 4601 Urban & Regional Economics

Lecture 2: Budget Constraint

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7 Summary
How does the price increase in gasoline affect your decision on commuting?

How is the interest rate related to housing?

Consumer theory predicts how consumers choose their bundle of goods under constraints.

In principle, consumers choose the best bundles of goods they can afford.

I’d like to have 5000 cheesecakes and 8000 cups of tea.

I’d rather have 3 cheesecakes and 4 cups of tea than 8 cheesecakes and no tea.

It seems like you consider two things when making purchases:

1. Your $ (this lecture)
2. Your 😊 (Lecture 3 & 4)

Do not mix them up. 😊 part alone doesn’t predict the consumer’s choice.
A *consumption bundle* contains $x_1$ units of commodity 1, $x_2$ units of commodity 2. It is denoted by

$$x = (x_1, x_2).$$

- $x = (2, 5)$ denotes a bundle of 2 cheesecakes and 5 teas.
- Commodity prices are denoted

$$p = (p_1, p_2).$$

- $p = (3, 1)$ means cheesecakes and tea are sold at $3 and $1 each.
- *Expenditure* on $x = (2, 5)$ is

$$p_1x_1 + p_2x_2 = 3 \times 2 + 1 \times 5 = 11.$$
A collection of bundles that are affordable forms the consumer’s budget constraint. A bundle \( x \) satisfies your budget constraint if

\[
p_1x_1 + p_2x_2 \leq m.
\]

your expenditure on \( x \) your income

A budget line is a collection of bundles \( x \) that are just affordable, i.e.,

\[
p_1x_1 + p_2x_2 = m.
\]

Suppose \((p_C, p_T) = (3, 2)\) and your income is $12.

What is your budget line?

\[
\frac{3x_C + 2x_T}{x} = \frac{12}{m}.
\]

Are the following on your budget line?

A. \((x_C, x_T) = (2, 3)\).
B. \((x_C, x_T) = (3, 3)\).
C. \((x_C, x_T) = (2, 2)\).
D. \((x_C, x_T) = (4, 0)\).

It is not practical to list all the affordable bundles one by one.
Rewrite (1) as

\[ x_T = \frac{-3}{2} x_C + 6. \]

Recall

\[ y = a \cdot x + b. \]

Then,

\[ x_T = \frac{-3}{2} x_C + 6. \]

- We can represent the budget line on the x-y plane.
- Continuity assumption: We assume that commodities are sold in any measure, i.e., \( x_1 \) is not necessarily a whole number. Tea is sold by ounces rather than by cups.
1. Bundles inside of the triangle satisfies the budget constraint.
2. Bundles on the slope is just affordable.
3. Bundles off the triangle is not affordable.

Budget Set and Constraint for Two Commodities

\[ p_1 x_1 + p_2 x_2 = m. \]

- **y-intercept** denotes the quantity of good 2 when you do not consume good 1 \( \left( \frac{m}{p_2} \right) \).
- **x-intercept** denotes the quantity of good 1 when you do not consume good 2 \( \left( \frac{m}{p_1} \right) \).
Question
Does the slope of the budget line have any meaning?

Answer
The following are the same:
1. The slope of the budget constraint.
2. Relative price.
3. Opportunity cost.
Exercise

Draw the budget lines for:

1. \((p_C, p_T) = (3, 2)\) and \(m = 12\).
2. \((p_C, p_T) = (6, 2)\) and \(m = 12\).
3. \((p_C, p_T) = (3, 1)\) and \(m = 12\).

How is the slope related to the prices?
The slope \( -\frac{p_C}{p_T} \) becomes steeper when
- \( p_C \) ↑.
- \( p_T \) ↓.

\( p_C \) and \( p_T \) per se do not determine the slope.

The price ratio determines what is affordable.

The price ratio has two interpretations:
- Relative price (market rate of exchange).
- Opportunity cost.

If you sell one slice of cheesecake, then you’ll receive \( p_C \) dollars. With \( p_C \) dollars, you can buy \( \frac{p_C}{p_T} \) cups of tea at the market.

A slice of cheesecake buys \( \frac{p_C}{p_T} \) cups of tea.

The slope tells you the market rate of exchange between \( x_C \) and \( x_T \) a.k.a. relative price of tea in terms of cheesecakes.

Note that the relative price is pre-determined in the market and your preferences do not affect it.
Notes on the slope:
- When the slope is given by $\frac{a}{b}$, on the x-y plane you can find the slope by:
  1. moving towards the east by $b$ and then to the north by $a$ (or south if it’s negative).
  2. moving towards the east by 1 and then to the north by $\frac{a}{b}$ (or south if it’s negative).

Watch out for the Flipping Fallacy:
- It is always a good idea to specify the unit of measure.
- Units are defined consistently and you can always count on them.
- Here we have $p_C$ ($/cheesecake$) and $p_T$ ($/tea$).

\[ \frac{p_C($/cheesecake$)}{p_T($/tea$)} = \frac{p_C}{p_T} \left( \frac{\text{teas}}{\text{cheesecake}} \right). \]

- Faux pas:

\[ p_C($/cheesecake$) \times p_T($/tea$) = p_C p_T \left( \frac{\$^2}{\text{cheesecake} \times \text{tea}} \right). \]

(A dollar squared per cheesecake tea...)
Opportunity Cost

Opportunity cost

is the best alternative that we forgo, or give up, when we make a choice or a decision.

- When you buy one more slice of cheesecake, you have to give up consuming $\frac{p_C}{p_T}$ units of tea.
- (Again, we do not care why you would trade cheesecakes and tea here. It is simply a market matter but not your preference matter.)

To sum up, the slope of the budget line represents
- the relative price, and
- the opportunity cost of buying (consuming) one unit of $x_C$ (measured in terms of tea).

Note we do not care who the consumer is at this point. Prices are given in advance.
You are sent to an isolated island to study its economy. One islander, Greg, consumes cod fish and tea leaves ($x_C$ and $x_T$). He has 10 tea leaves and no cod fish.

You have noticed that 2 tea leaves are bartered for 6 oz of cod fish. Can you still draw Greg’s budget line without currency?

IMF has decided to introduce currency to the island. What do you think are the prices of cod fish and tea leaves going to be?
Budget line without currency?

- 2 tea leaves buys 6 oz of cod fish (market rate of exchange), i.e., 3 oz/tea leaf.
- Greg has 10 tea leaves.
- He can buy
  - \((x_C, x_T) = (0, 10)\).
  - \((x_C, x_T) = (6, 8)\).
  - \((x_C, x_T) = (12, 6)\).
  - \((x_C, x_T) = (18, 4)\).
  - \((x_C, x_T) = (24, 2)\).
  - \((x_C, x_T) = (30, 0)\).
You do not need to know individual prices to find budget constraints.
You only need:
- Consumer’s income.
- Relative price (i.e., market rate of exchange) of commodities.

What would the prices of cod fish and tea leaves be?
- Note the market rate of exchange (relative price) is 3 oz/tea leaf.
- There are many candidate pairs of prices:
  - \((p_C, p_T) = (6, 2)\),
  - \((p_C, p_T) = (3, 1)\),
  - \((p_C, p_T) = (150, 50)\),
  - \((p_C, p_T) = (1, 1/3)\),
- all of which yields the same relative price of \(p_C/p_T = 3\) (tea leaves/oz).
- Greg’s budget constraint is independent of how we assign the prices:
  \[
  6x_C + 2x_T = 2 \times 10 \quad \Rightarrow \quad x_T = -3x_C + 10
  \]
  \[
  150x_C + 50x_T = 50 \times 10 \quad \Rightarrow \quad x_T = -3x_C + 10.
  \]
- Having two prices is redundant in that we only need the relative prices to specify the slope.
Numéraire Good

is a commodity whose price is normalized to one.

- Numéraire means "unit of account".
- Any commodity can be chosen as the numéraire without changing the budget set or the budget constraint.
- In the previous example, we can chose tea leaves to be a numéraire.
- Interpret: consider tea leaves to be the island’s dollar bill.
- We can reduce the number of parameters in this manner.

When you buy a house, do you really think of cheesecakes that you have to give up?
- Rather, we just think of "other things" that we could buy if we did not buy a house.
- Economists have a nice hypothetical commodity called
Composite Good

is a basket of commodities consisting of goods other than a good in question (a house for example).

- A composite good $x_2$ is a basket of one cheesecake, two cups of tea, a pair of shoes, your phone bill, etc.
- A price of composite good $p_2$ is a cost of purchasing one basket.

Your budget constraint is

\[ p_1 x_1 + p_2 x_2 \leq m \]

expenditure on a house exp. on other goods

rather than

\[ p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots + p_N x_N \leq m. \]
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### Trinity
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### Some Useful Assumptions
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### Applications
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- Fisher Equation

### Summary

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#### Change in Income

**Q: How does the budget line change when prices and income change?**

1. Stay put
2. Parallel shift
3. Rotation (pivoting)
4. Combination of parallel shift and rotation
Suppose the income increases from 1 to 100:

\[ p_1x_1 + p_2x_2 = 1 \]

\[ \downarrow \]

\[ p_1x_1 + p_2x_2 = 100. \]

- The intercept changes. Increased income allows you to purchase more.
- The slope remains constant. Your income doesn’t affect the market rate of exchange (relative price).

\[ x_2 = \frac{-p_1}{p_2}x_1 + \frac{1}{p_2} \rightarrow x_2 = \frac{-p_1}{p_2}x_1 + \frac{100}{p_2}. \]

- A change in income results in a parallel shift.
Suppose $p_1$ increases from 2 to 100:

\[2x_1 + p_2 x_2 = m\]
\[\downarrow\]
\[100x_1 + p_2 x_2 = m.\]

- The slope changes. The market exchange rate reflects the increase.
- The horizontal intercept changes. You can buy less $x_1$.
- The vertical intercept remains constant. Increase in $p_1$ does not affect the maximum amount of $x_2$ you can buy.
- A change in price results in rotation.

\[x_2 = \frac{-2}{p_2} x_1 + \frac{m}{p_2} \rightarrow x_2 = \frac{-100}{p_2} x_1 + \frac{m}{p_2}.\]
Exercise

Greg spends his income on cheesecakes ($x_C$) and tea ($x_T$). The price is given by $(p_C, p_T) = (3, 2)$.

1. Draw Greg’s budget line.

2. What happens to the budget line if $p_T$ drops to 1? In particular, consider the change in
   1. maximum amount of cheesecakes he can purchase
   2. maximum amount of tea he can purchase
   3. his opportunity cost of a cheesecake in terms of tea

3. Can you say the rotation of budget line leave Greg happier than before? If not, what additional information do you need?
While the reduction in tea’s price relaxes Greg’s budget constraint, it does not necessarily increase his happiness.

- He may not care for the tea in the first place.
- We also need to know Greg’s preferences as well as his budget constraint to understand his consumption behavior (to be discussed in Lecture 3 & 4).
The budget constraint can represent a wide range of commodities.

Examples include budget constraints over time and savings.
Greg’s monthly income is $4,000 this month and $0 in July.
He will take a month off to spend July in Cancun for his summer vacation.
He is planning to save some of his income this month for his trip next month.
Monthly interest rate is 10%.

Q1: How can we represent his bundle? What would be $x_C$ and $x_T$ in this case?
Note we are not concerned with cheesecakes this month or tea next month.
Rather, we are concerned with what Greg can spend this month and next month.
A1: Represent this month’s consumption by a composite commodity $x_6$ and next month $x_7$.
   June’s basket ($x_6$) might include cheese burgers, pens, gasoline, and
   July’s basket ($x_7$) might include mojitos, sunscreens and sombreros,
   which we do not care about. We’d like to see the relationship betw. affordability and savings.
Let $p_6 = p_7 = 1$ for simplicity (i.e., both commodities are numéraire).

Some Experiments

Take $x_6$ on x-axis and $x_7$ on y-axis. Where do the following situations appear on your graph?

1. The bundle $(x_6, x_7)$ when Greg doesn’t save at all.
2. The bundle when Greg saves all of his income for next month.
3. The bundle when Greg uses $1,500 this month and save the rest for next month.
4. The bundle when Greg uses $2,000 this month and save the rest for next month.
It looks like the bundles are on the same line. They are:

$$x_7 = (1 + .1)(2000 - x_6),$$

consumption in July  interest  savings

or equivalently,

$$x_7 = -1.1x_6 + 4400.$$
• Recall the trinity.
• The opportunity of $x_7$ is 1.1, i.e., if Greg gives up one unit of $x_6$, then he will get 1.1 units of $x_7$ in July.

In the previous example, the value of a basket does not change over time.
Reason: the price does not change over time.
What happens if the prices change from month to month?
In particular, how is Greg’s purchasing power affected by the change in prices?
To keep things simple, suppose there is one commodity in the world: tea leaves.
Consider two periods 1 and 2.
We have a bundle \((x_1, x_2)\) and price \((p_1, p_2)\).

**Definitions**

- **Inflation rate** \(\pi\): measures the growth in prices:
  \[
  1 + \pi = \frac{p_2}{p_1}.
  \]
  (2)

- **Nominal interest rate** \(i\): if you save a dollar in period 1, you’ll receive \(1 + i\) dollars in period 2. Likewise, to receive a dollar in period 2, you need to save \(\frac{1}{1+i}\) dollars in period 1.

If you save \(p_1\) dollars, you’ll get \((1 + i)p_1\) dollars in period 2.
That doesn’t imply you can simply purchase \(1 + i\) tea leaves in the next period.
Consider the following to track down the change in Greg’s purchasing power:

1. To buy 1 leaf in period 1, Greg needs $p_1$. i.e., the purchasing power of $p_1$ is 1 leaf.
2. $p_1$ does not guarantee the same purchasing power in period 2.
3. $p_1$ translates to $p_1(1 + i)$ in period 2.
4. How many tea leaves does $p_1(1 + i)$ buy?

$$p_1(1 + i) \frac{1}{p_2} = \frac{p_1(1 + i)}{p_2},$$

5. The ability to buy 1 leaf in period 1 translates to the ability to buy $\frac{p_1(1 + i)}{p_2}$ leaf in period 2.

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Real Interest Rate

Real interest rate $r$ measures the growth in purchasing power:

$$1 + r = \frac{p_1(1 + i)}{p_2}. \quad (3)$$

- Recall (2): $1 + \pi = \frac{p_2}{p_1}$. Then we can write (3) as

$$1 + r = \frac{1 + i}{1 + \pi},$$

or in other words,

$$(\text{change in purchasing power}) \times (\text{inflation rate}) = (\text{nominal interest rate})$$

- The relationship above is called Fisher equation.
If the inflation rate is low while the nominal interest rate is high, then the real interest rate (growth rate in purchasing power) is high.

i.e., it takes more money to maintain the purchasing power over time.

Example

Compute the real interest rate in Greg’s Cancun example. Recall

- nominal interest rate is \( i = .1 \).
- \( p_6 = p_7 = 1 \).

Example Question

Greg consumes a composite good \( x_6 \) this month and \( x_7 \) next month. The prices of the composite goods are normalized to unity in both months. His monthly income is $4,000 this month and $0 in July as he will take a month off to spend July in Tahiti for his summer vacation.

1. Suppose he saves $1,500 in this month for his trip next month. How much \( x_6 \) can he buy this month?
2. Suppose the monthly interest rate is 5%. How much \( x_7 \) can he buy next month?
3. Take \( x_6 \) on the x-axis and \( x_7 \) on the y-axis and plot the following bundles on your graph:
   - The bundle \((x_6, x_7)\) when Greg does not save this month.
   - The bundle when he saves all of his income for next month.
   - The bundle described in questions 1 and 2.
   - The bundle when he saves $2,000.
4. What is the opportunity cost of \( x_6 \) in terms of \( x_7 \)?
5. Suppose \( p_7 = 2 \) instead of \( p_7 = 1 \). What is the real interest rate? Does Greg’s purchasing power increase or decrease after the change? Interpret.
- Represent what is affordable to a consumer.
- Trinity.
- Changes in the budget line associated with the changes in parameters.
- Describe intertemporal budget constraint.
- Relationship among prices, interest rates, inflation rates and purchasing power.