Chapter 5: Utility Maximization Problem

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Introduction

Consumer Theory Overview

- What do consumers face?
  - Chapter 2 & 10
- What do consumers want?
  - Chapter 3 & 4
- How do consumers resolve conflict above?
  - Chapter 5

Consumers choose the best combination of goods they can afford. The framework to analyze their choice behavior is called utility maximization problem (UMP).

**Definition 1.1 (Utility Maximization Problem)**

Liz chooses the bundle \((x_C, x_T)\) that gives her the highest utility level among the affordable bundles. In other words, she

\[
\max u(x_C, x_T) \text{ subject to } p_C x_C + p_T x_T = m.
\]
Introduction

2 Solving UMP
   1. Budget Line Meets Indifference Curves
   2. Tangency
   3. To Find the Exact Solution

3 Extreme Cases

Now We Know

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How exactly do we find Liz’s optimal bundle $x^* = (x_C^*, x_T^*)$?

Consider a case when
   - Utility function $u(x_C, x_T) = x_C x_T$
   - Income $m = 60$
   - Price $(p_C, p_T) = (3, 2)$

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Tangency

Fact 2.1 (Tangency Condition)

For standard preferences*, the indifference curve is tangent to the budget line at the optimal bundle, i.e., they share the same slope at $x^*$.  

*This condition does not apply to certain types of preferences. See Exercise 3.2 for example.
What does tangency condition imply?

- Trinity on the budget side
- Trinity on the preference side
- Liz's idea of the tea's worth coincides with market's idea of the tea's worth when she chooses the right amount.

What if tangency condition is not met?
To Find the Exact Solution

Where exactly is the budget line tangent to the indifferent curve?
- We need two conditions to find the point of tangency:

**Fact 2.2 (Finding the Solution)**
For standard utility functions, the solution can be found using
- tangency condition and
- budget constraint

**Problem 2.3 (UMP)**
max_{x_C, x_T} \left( x_C x_T \right) \text{ subject to } 3x_C + 2x_T = 60.
- MRS at \((x_C, x_T)\) is given by \(\frac{-x_T}{x_C}.\)
- The relative price is \(\frac{-3}{2}.\)

In what follows, I’ll get you MRS except for perfect substitutes.

**Tangency condition:**
\[
\frac{-x_T}{x_C} = \frac{-3}{2} \Rightarrow x_T = \frac{3}{2} x_C. \tag{1}
\]
- There are lots of bundles \((x_C, x_T)\) satisfying (1).
To Find the Exact Solution

To pin down the solution, use budget constraint:

\[ 60 = 3x_C + 2 \frac{3}{2} x_C. \]

Conclude \( x^* = (10, 15). \)

Exercise 2.4 (UMP)

Liz spends her income ($5) on coins (\( x_C \)) and tea (\( x_T \)). Price is given by \( (p_C, p_T) = (2, 1) \). His preferences are represented by \( u(x_C, x_T) = x_C + \sqrt{x_T}. \)

- Write Liz's utility maximization problem.
- What is the tangency condition in this example? (MRS at \( (x_C, x_T) \) is \(-2 \sqrt{x_T}\)).
- Which bundle will Liz choose?
To Find the Exact Solution

- Indifference Curves
- Budget Line

When Tangency Condition Does Not Hold

- There are some exceptions where tangency condition does not apply.
Perfect Substitutes

- Consider six-packs and bottles of Corona \((x_6, x_1)\).
- Liz's MRS is 6 everywhere (why?)
- Consider three cases:
  1. Market rate of exchange is higher than 6.
  2. Market rate of exchange is lower than 6.

Let's say the market rate of exchange is larger than 6, say 12.
Liz will spend all her income on bottles.
This type of solution is called a corner solution (e.g., \((8, 0), (35, 0), (0, 68)\) etc.)
If the market rate of exchange is lower than 6...
Perfect Substitutes

Market rate of exchange is exactly 6...

Exercise 3.1 (Perfect Substitutes)

Liz has $12 and spend his income on Coke & Pepsi ($C, $P$). Suppose Coke is sold for $4 while Pepsi is sold for $2.

Find Liz’s optimal bundle.

Confirm your answer using indifference curves and the budget line.
Perfect Substitutes

For perfect substitutes, MRS is not defined.
What to do with them then?
Consider left gloves ($x_L$) and right gloves ($x_R$).

Perfect Complements

For perfect complements, MRS is not defined.
What to do with them then?
Consider left gloves ($x_L$) and right gloves ($x_R$).
Perfect Complements

Optimal bundle $x^*$ for left gloves and right gloves is

- on the budget line and
- on the 45 degree line.

Exercise 3.2 (Perfect Complements)

Consider a bundle $(cereal, milk) = (x_C, x_M)$.

- Liz says he can’t have cereals without milk and the only time she drinks milk is when she eats his cereals.
- Liz’s preferred cereal-milk ratio is 2 to 3.
- $(p_C, p_M) = (6, 4)$.
- $m = 24$.

- Draw her budget line.
- Draw indifference curves at $(2, 3)$ and $(4, 6)$.
- Mark the bundle Liz will choose on your graph.
Perfect Complements

- Liz’s preferred bundles are lined up on the ray $x_M = \frac{3}{2}x_C$.
- The optimal bundle is
  - on the ray $x_M = \frac{3}{2}x_C$ and
  - on the budget line $x_M = \frac{3}{2}x_C + 6$.

- How to set up the utility maximization problem.
- Tangency condition for standard cases.
- Optimal bundles for extreme preferences.
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