

Econ 3023 Microeconomic Analysis

SUGGESTED SOLUTION #3

Instructor: Hiroki Watanabe

Fall 2012

1 Cobb-Douglas Production Function

A. (a) \( f(x) = f(x_C, x_K) = f(4, 5) = 4 \cdot 5^2 = 100. \)

(b) \( f(x) = f(x_C, x_K) = f(5, 5) = 5 \cdot 5^2 = 125. \)

(c) \( f(x) = f(x_C, x_K) = f(8, 5) = 8 \cdot 5^2 = 200. \)

(d) \( f(x) = f(x_C, x_K) = f(9, 5) = 9 \cdot 5^2 = 225. \)

B. (a) \( MP_C(x_C = 5, x_K = 5) = f(x_C = 5, x_K = 5) - f(x_C = 4, x_K = 5) = 125 - 100 = 25. \)

(b) \( MP_C(x_C = 9, x_K = 5) = f(x_C = 9, x_K = 5) - f(x_C = 8, x_K = 5) = 225 - 200 = 25. \)

Chefs do not exhibit diminishing marginal product. The number of additional cheesecakes does not change (it is always 25 cheesecakes) no matter how many chefs are on payroll with Mike’s technology. 1

C. (a) \( (x_C, x_K, y) = (4, 5, 20) \) is feasible: \( f(x_C = 4, x_K = 5) = 4 \cdot 5^2 = 100 > 20 = y. \)

(b) \( (x_C, x_K, y) = (4, 5, 100) \) is feasible (and efficient): \( f(4, 5) = 100 = y. \)

(c) \( (x_C, x_K, y) = (4, 5, 101) \) is not feasible: \( f(4, 5) = 100 < 101 = y. \)

(d) \( (x_C, x_K, y) = (8, 10, 200) \) is feasible: \( f(8, 10) = 800 > 200 = y. \)

D. (a) \( (x_C, x_K, y) = (4, 5, 20) \) earns him \( \pi(x_C, x_K, y) = py - w_C x_C - w_K x_K = 4 \cdot 20 - 2 \cdot 5 - 3 \cdot 5 = 55 \) dollars.

(b) \( (x_C, x_K, y) = (4, 5, 100) \) earns him \( \pi(4, 5, 100) = 377 \) dollars.

(c) \( (x_C, x_K, y) = (4, 5, 101) \) earns him \( \pi(4, 5, 101) = 381 \) dollars.

(d) \( (x_C, x_K, y) = (8, 10, 200) \) earns him \( \pi(8, 10, 200) = 754 \) dollars.

E. Increasing returns to scale:

\[
 f(2x) = (2x_C)(2x_K)^2 = 8x_C x_K^2 = 8f(x_C, x_K) > 2f(x_C, x_K).
\]

2

2 Marginal Product & Returns to Scale

A. See Figure 1.

---

1 Department of Economics, Labovitz School of Business & Economics, University of Minnesota Duluth (watanabe@d.umn.edu).

2 For advanced students only: Note

\[
 MP_C(x_C, x_K) := \frac{\partial f(x_C, x_K)}{\partial x_C} = x_C^2 = 25
\]

for any \( x_C > 0. \)

2 For advanced students only: In general, a Cobb-Douglas production function \( f(x) = x_C^a x_K^b \) exhibits

(a) IRS if \( a + b > 1 \) (this question)

(b) CRS if \( a + b = 1 \)

(c) DRS if \( 0 < a + b < 1. \)

Proof is pretty straightforward. See it for yourself.
B. See Table 1. Alex exhibits diminishing marginal product as his marginal product decreases with time.

<table>
<thead>
<tr>
<th></th>
<th>Alex</th>
<th>Leah</th>
</tr>
</thead>
<tbody>
<tr>
<td>During 3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>After</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1.

C. [Optional] His technology exhibits locally constant returns to scale (CRS) and locally decreasing returns to scale (DRS). At $x = 1$, his production function exhibits CRS:

$$f(2 \cdot 1) = 2(2 \cdot 1) = 2f(1).$$

At $x = 4$, his technology turns into DRS:

$$f(2 \cdot 4) = (2 \cdot 4) + 3 = 11 < 2f(4) = 14.$$

D. [Optional] No. After 3 hours, his technology turns DRS. He would want to split his workload so that he can call it a day before he reaches the DRS region. For example, if I gave Alex $y = 12$ problems, he’ll need to work for 3 hours on Monday and Tuesday (6 hours in total), whereas he’ll need to work for as many as 9 hours if he went out on Monday and work on the homework only on Tuesday.  

3 Short-Run & Long-Run Production Function

A. 1 cheesecake per an additional lefty.

B. 2 cheesecakes per an additional righty.  

3A note to 4935 folks: Replace time with population in this example and you’ll find yourself in the urban economics world. You would want to split the city in two if there are too many people, just like working more than 3 hours straight is not efficient. DRS is a centrifugal force.

4For advanced students only: Once again, note

$$MP_L(x_1, x_R) := \frac{\partial f(x_1, x_R)}{\partial x_L} = 1, \quad \text{and} \quad MP_R(x_1, x_R) := \frac{\partial f(x_1, x_R)}{\partial x_R} = 2.$$
C. CRS:
\[ f(2x) = f(2x_L, 2x_R) = (2x_L) + 2(2x_R) = 2(x_L + 2x_R) = 2f(x_L, x_R). \]

D. The input bundles \( x = (x_L, x_R) \) that satisfy the relationship \( x_L + 2x_R = 10 \) constitute the isoquant at \( y = 10 \). Write \( x_R \) by itself: \( x_R = \frac{-1}{2} x_L + 5 \). See Figure 2(a) or 2(b) (the blue line is the isoquant at \( y = 10 \)).

E. The slope measures the marginal rate of technical substitution (recall trinity on the production side). Cory can safely replace one lefty with .5 righty without affecting the ongoing production level. He will loose 1 cheesecake by letting go one lefty but .5 righty will produce 1 cheesecake to cover up the loss.

F. See Figure 2(c).

G. It shifts the production function upwards. See Figure 2(c). An additional left-handed chef simply adds two more cheesecakes for any given \( x_R \). Note that added left-handed chefs do not improve or worsen the right-handed chef’s marginal product. Two types of chefs are independent from each other. They are perfect substitutes in Cory’s bakery. Compare this to a Cobb-Douglas production function (p.56 in Chapter 18). Short-run production function (Figure 2(c)) is a slice of long-run production function (Figure 2(d)) along \( x_L = 2 \) (the blue line) and \( x_L = 4 \) (the red line). In a Cobb-Douglas production function on p.56, added kitchen equipment increases the marginal productivity of the chefs, i.e., chefs become more productive with a help of advanced kitchen equipment.
equipment. They can let the machine beat the cream cheese and do something else that they are good at. This type of synergy does not take place in Cory’s bakery.

4 Utility / Profit Maximization and Efficiency

A. Czar’s involvement is not necessary because the tangency condition spontaneously leads to the efficient outcome. Efficiency is measured in terms of the profit that each farmer can earn. If the marginal cost is higher than the ongoing price, a farmer will (without czar’s involvement) down scale their operation because additional output incurs more loss than the revenue. If the marginal cost is lower than the ongoing price, a farmer will (once again, without czar’s involvement) expand their operation because additional output brings in more revenue than the marginal cost of production. In the end, all the farmers find the scale of operation in consultation with the tangency condition to maximize profit. The shared price will let productive farmers (the ones with slow growth in marginal product) produce more and less productive farmers limit their scale of production as their cost of production increases fast and marginal cost reaches the price early on. Efficiency is already ingrained in farmer’s decision making, and in particular in their tangency condition, and interruption will only result in efficiency loss by reducing profit of at least one of the farmers.

B. [Optional] Without cheesecake market, Liz will lose the opportunity to earn her consumer surplus. Suppose she is willing to pay 10 cups of tea (a numéraire) for the first slice and the cheesecake is sold for $5. She will earn $5 of consumer surplus, which disappears if the market disappears (a deadweight loss). Consequently, she will be forced to select a corner solution, where she can only consume tea and no cheesecakes. If her preferences are (strictly) concave, the corner solution is not the optimal bundle and she will end up with a smaller utility level than what she could have achieved with the cheesecake market.