Math 3280 Practice Midterm 2

The test will primarily cover chapters 4, 5, and 6, although some material from earlier chapters might be involved. The actual midterm will have 3 or 4 required questions. One sheet of notes and a calculator are allowed - however you must indicate the use of a calculator, and you must show the steps in your calculations for full credit.

(1) Find the general solution to the ODE: $y^{(3)} - 5y'' + 12y' - 8y = 0$.

(2) Find the solution to the initial value problem $y'' - 2y' + 5y = e^{2x}$, $y'(0) = 0$, $y(0) = -1$.

(3) Write down the form of a particular solution y_p of the ODE $y'' + y = x^2 e^x + \cos(x)$. You do not have to determine the coefficients of the functions.

(4) If an $n \times n$ matrix A has the property that $A^3 = 2A$, what are the possible values of the determinant of A?

(5) Solve the initial value problem $y''' - 27y = e^{3x}$, $y(0) = y'(0) = y''(0) = 0$.

(6) Find a basis for the subspace S of solutions to the system within the vector space $\{(x_1, x_2, x_3, x_4, x_5, x_6) |\ x_i \in \mathbb{R}\} = \mathbb{R}^6$:

$$x_1 - x_2 + x_4 + 4x_5 = 0$$
$$x_1 + x_2 + x_4 + 4x_5 + x_6 = 0$$

(Your answer should be a set of 6-dimensional vectors.)

(7) Rewrite the initial value problem $y''' + y'' + y = t$, $y(0) = y'(0) = y''(0) = 0$ as an equivalent first-order system.

(8) The matrix

$$A = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

where a and b are real numbers, is diagonalizable, i.e. there exists a matrix P such that $P^{-1}AP = D$ where D is diagonal. Compute D.

(9) Indicate whether each of the following statements is true or false.

(a) The set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x + y + z = 0$ is a vector subspace of $\mathbb{R}^3$ of dimension 2.

(b) The set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x + y = 1$ is a vector subspace of $\mathbb{R}^3$ of dimension 2.

(c) The set of solutions to the differential equation $y'' + xy' + x^2y = 0$ is a vector space of dimension 2.

(d) The set of solutions $(x, y, z) \in \mathbb{R}^3$ of the system below is a vector subspace of $\mathbb{R}^3$ of dimension 1.

$$\begin{aligned} x + 2y + 3z &= 0 \\ 4x + 5y + 6z &= 0 \\ 7x + 8y + 9z &= 0 \end{aligned}$$

(e) The polynomials $1+x$, $1-x$, $1+x^2$ are a basis for the vector space of polynomials with real coefficients of degree less than or equal to 2.