On a conjecture of Marimuthu et al.

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Abstract

For a given digraph $D = (V, A)$ with $|V| = p, |A| = q$, bijection $f : V \cup A \to \{1, 2, \ldots, p + q\}$ such that $f(V) = \{1, 2, \ldots, p\}$ is called a V-super vertex in-antimagic total (V-SVIAMT) labeling of D if the vertex weights $w(x) = f(x) + \sum_{yx \in A} f(yx)$ are all different. Marimuthu et al. [1] conjectured that all digraphs allow a V-SVIAMT labeling. We prove their conjecture.

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1 Proof of the conjecture

Let $D = (V, A)$ be a digraph with $V = \{v_1, v_2, \ldots, v_p\}$ and for $i = 1, 2, \ldots, p$ let $A_i = \{v_t v_i \mid v_t v_i \in A\}$ be the set of incoming arrows for vertex v_i. Let $|A_i| = d_i$. Then $A = \cup_{i=1}^{p} A_i$ and $|A| = q = d_1 + d_2 + \cdots + d_p$.

Let f be a bijection $f : V \cup A \to \{1, 2, \ldots, p+q\}$ such that $f(V) = \{1, 2, \ldots, p\}$. For $v_i \in V$, let $w(v_i) = f(v_i) + \sum_{v_j v_i \in A_i} f(v_j v_i)$. If the values of $w(v_i)$ are distinct for all vertices in V, then f is called a *V-super vertex in-antimagic total (V-SVIAMT) labeling* of D.

Conjecture 1 (Marimuthu et al., [1])**.** *All digraphs allow a V-SVIAMT labeling.*

Marimuthu et al. remarked that the following conjecture "seems to be difficult to solve." We prove the conjecture. In $D = (V, A)$, we assume without loss of generality that $d_1 \leq d_2 \leq \cdots \leq d_p$. For each v_i we arrange the tail vertices of all d_i incoming arrows. Let S_i be the set of their subscripts. Thus, $S_i = \{i_j \mid v_{i_j} v_i \in A_i, 1 \leq j \leq d_i\}$ for $1 \leq i \leq p$. For vertices with zero in-degree is $S_i = \emptyset$. We adopt the common fact that $\sum_{m=1}^{0} d_m = 0$.

Theorem 2. *All digraphs allow a V-SVIAMT labeling.*

Proof. Set $f(v_i) = i$ for $i = 1, 2, \ldots, p$ and $f(v_{i_j} v_i) = p + j + \sum_{m=1}^{i-1} d_m$ for $i = 1, 2, \ldots, p$ and $i_j \in S_i$. Clearly, the sequence $w(v_1), w(v_2), \ldots, w(v_p)$ is strictly increasing and therefore D allows a V-SVIAMT labeling. □

We remark that a slightly modified technique can be used to prove similar results for other types of antimagic total labelings for both directed and undirected graphs.

References

[1] G. Marimuthu, M.S. Raja Durga, G. Durga Devi, *V-super vertex in-antimagic total labelings of digraphs*, J. Graph Labeling **1 (1)**, 2015, 21–30.