1. Visualizing the three-body problem. The problem of characterizing the dynamics of three celestial bodies (modeled as point masses) interacting according to Newton's law of gravitation is an old and famous one; there is a mind-boggling intricate structure to the possibilities. One way to get a better handle on the problem is to think of the trajectory as a path in "shape space", the space of triangles, which can be visualized as a sphere or plane. It would be very useful to have a program which showed the path in shape space for three bodies.

   Required background: Good programming skills (ideally some Java experience), Math 3280 and 3298 or equivalent.

2. Real conics tangent to circles. Typically in algebra and algebraic geometry, asking questions about real solutions is much harder than those over the complex field. In 1864, Chasles showed that given five conics (curves of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$) there are 3264 (in general complex) conics which are tangent to the given five. In 1997, Ronga, Tognoli, and Vust proved that all 3264 of the tangent conics can be real. Their proof, and many other results of this type, relied on knowledge of the case when the tangent conics are all straight lines through the points of a regular pentagon; the results only apply to conics which are deformations of this configuration. It would be interesting to see what happens far away from this case, such as when the given conics are circles on a regular pentagon. A first step would be studying some specific cases numerically.

   Required background: Math 3298 and some facility in a mathematical programming environment such as Mathematica or Maple; some knowledge of abstract algebra would be helpful.

3. Newton polyhedra and the mixed volume of polynomial systems. Given a system of polynomial equations in n variables, we can study some properties of the solutions by simply considering the Newton polytopes of the equations. Loosely speaking, the Newton polytope of a polynomial is formed by the exponents of each term of the polynomial (more precisely by the convex hull). For example, the Newton polytope of the equation $1+3x+4y+2xy$ is the square formed by the points $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$, which are the pairs of exponents of each term. The Newton polytopes do not depend on the coefficients of each term - in the previous example, we would get the same square Newton polytope from the equation $1 + x + y + xy = 0$. The mixed volume of a system of equations only depends on the Newton polytopes, and gives an upper bound on certain types of solutions to the system. This project would investigate how the mixed volume depends on the equations we pick for the system, for particular systems coming from celestial mechanics or vortex dynamics.

   Required background: Math 3298 and some facility in a mathematical programming environment such as Mathematica or Maple. Knowledge of linear programming would be extremely helpful but not necessary. Some familiarity with Unix/Linux and/or abstract algebra would also help.