Real Analysis, Math 8201

List of Possible Problems for Midterm 2 Test

(Fri. May 1, 2015, 12-2 or by arrangement)

April 20, 2015DRAFT

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Be able to state and apply the following definitions:

1. Outer and inner Lebesgue measure, including all the definitions of the measures of special rectangles, special polygons, open sets, and compact sets.
2. Lebesgue measurable sets in terms of the outer and inner measures of the set. Be sure to include the case when the outer measure is infinite.
3. A set that is not Legbesgue-measurable. Be able to write down such a set. (You are not required to prove that it is not measurable.
4. Fat Cantor sets
5. The Cantor-Lebesgue function.
6. An algebra and a σ-algebra.
7. The Borel σ-algebra. Know how to describe a set that is Lebesgue measurable, but not Borel measurable.
8. A null set.
9. A Lebesgue-measurable function – five equivalent definitions.
10. A simple function.
11. Class S of simple functions.
12. $\int sd\lambda$ for $s \in S$.
13. $\int fd\lambda$ for f measureable, nonnegative.
14. L^1, L^1 norm, convergence in L^1
15. f_-, f_+.
16. a.e.
17. Integrals over subsets of $\Re^n$.
18. Measure space (X, M, μ)
19. Step function on I
20. Riemann integral on I.
21. Lower and upper semicontinuous
22. $\underline{f}, \overline{f}$.
23. f_y, A_y

Be able to state the following results. Proofs of those with a 'p' should also be known. The lists of 'properties' below need not be memorized per se, but would be more likely to appear as true-false questions. Properties to be proved will be explicitly stated.

1. Properties $O1^p, O2^p, O3^p, O4^p, O5, O6$
2. Properties $C1^p, C2^p, C3^p, C4$
3. Properties $*1^p, *2^p, *3^p, *4^p, *5$
4. Theorem on Approximation (for sets in L_0 p. 45) and its Corollary (p. 46)
5. Theorem on Countable Additivity (for sets in L_0 p. 47)
6. Properties M1 – M4,M5^p, M6 – M10
7. $\lambda^*(TA) = |\det(T)|\lambda^*(A)$, $\lambda^*(z + A) = \lambda^*(A)$, $\lambda^*(tA) = t^n\lambda^*(A)$, and the analogues for inner measure and measure.
8. LICT
9. Fatou's Lemma^p (assuming LICT)
10. LDCT
11. The two approximation theorems of functions in L^1 by functions in C_c and C_c^∞.
12. The two Fubini theorems

Simple proofs that were part of previously done homework problems, or a step in such a proof, or a similar step or proof might also be asked. In particular,

- Ch 1: 2g,10,18abc,19,43,44 (not originally assigned)
- Ch 2: 3 (OK to assume the only open and closed sets of $\Re^n$ are the empty set and $\Re^n$), 12, 31
- Ch 5: 2,6, 16ab, 21
- Ch 6: 1,5,9
- Ch 7: 1,2,22
- Ch 8: –

Examples:

1. A function that is Lebesgue integrable, but not Riemann integrable
2. A sequence of integrable functions $\{f_k\}$ with the property that $\int \lim f_k d\lambda \neq \lim \int f_k d\lambda$.
3. An example of a sequence of functions $\{f_k\}$ and a function f for which f_k converges to f in L^1, but does not converge pointwise.
4. A set A measurable in $\Re^n$ but A_y is not measureable in $\Re^m$ $(n = l + m)$ for at least one y.