

## MIDTERM TOPIC LIST

Dynamical Systems

Math 5260

Bruce Peckham

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For midterm on Friday, Oct. 18, 2013: 9:20 - 10:50 or 10 - 11:30am

In general, the midterm will cover any topics we covered in Chapters 1-12. The focus will be on “basic” material. Homework type questions from previous assignments will be emphasized. I will attempt to make the problems noncomputationally intensive. The following list of topics should give you a more specific idea of what kinds of questions will be asked.

### 1. Definitions to know:

- Fixed point, periodic point, orbit, cycle, period, prime or least period
- Attracting, superattracting, repelling, (linearly) neutral periodic point (orbit, cycle), weakly attracting, weakly repelling; use of chain rule in determining these adjectives
- Phase portrait
- Graphical analysis
- Discrete dynamical system vs. continuous dynamical system
- Bifurcation, incl. esp. saddle-node and period doubling (nondegeneracy conditions not necessary to memorize)
- The sequence space  $\Sigma$ , the “usual” metric on  $\Sigma$ , the shift map  $\sigma$ .
- Three properties of a chaotic system and all terminology used in the def. of the properties.
- $A$  dense in  $B$  for  $A \subset B$ .
- Homeomorphism, topological conjugacy, topological semiconjugacy

### 2. Results to know:

- Relationship between the shift map on the symbol sequence space and the quadratic map on the invariant Cantor set (for  $c < -2$ ).
- Sarkovskii’s theorem, including Sarkovskii’s ordering
- “Negative Schwarzian Derivative Property” (which is true for any quadratic): Any attracting periodic orbit must attract a critical point.

3. Techniques to know:

- Locating fixed and periodic pts/orbits analytically (for individual maps and for families of maps)
- Locating period- $n$  pts/orbits of  $f$  from graphs of  $f$  and  $f^n$ .
- Interpreting orbit diagrams (identifying, for example, parameter values corresponding to maps with attracting orbits of a certain period, or saddle-node bifurcations or period-doubling bifurcations)
- Determining stability of periodic orbits either analytically or from graphs.
- Constructing a graph of  $f$  so that  $f$  has a point  $p$  that is, for example, a period- $n$  point, and the derivative  $(f^n)'(p)$  has a specified value. (Hint: use the chain rule!)
- Constructing the graph of iterates of  $f$  given the graph of  $f$ .
- The construction of the invariant Cantor set  $\Lambda$  for  $x^2 + c$  with  $c < -2$ .
- The construction of an itinerary map.
- Given a map and a change of variables, find the equation for the map in the new variables.
- Recognizing a saddle-node and/or period doubling bifurcation from a sequence of graphs of a family of maps as a parameter changes
- Determining the number of prime periodic orbits of each period for  $x^2 - 2$  (equivalently  $2x \pmod{1}$ ,  $4x(1-x)$ ,  $\sigma$ )
- Determining the number of period- $n$  windows for each  $n$  in the orbit diagram for the family  $x^2 + c$ .
- Locating each period- $n$  window in the orbit diagram for the family  $x^2 + c$ .

4. Proofs to know:

- Prove that if  $f$  is continuous,  $f$  has a periodic point implies  $f$  has a fixed point.
- Prove either direction of the Proximity Theorem.
- Prove  $\sigma : \Sigma \rightarrow \Sigma$  is chaotic. (Prove any or all 3 properties.)

5. Anything else we've covered that I think is easy.