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Linear Algebra, Math 4326

Quiz 6, Spring 2016

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1. (2 pts) Let W be a vector subspace of $\mathbb{R}^n$. Define $W^\perp = \{\vec{v} \in \mathbb{R}^n : \vec{v}\cdot\vec{w} = 0 \; \forall \vec{w} \in W\}$. Show that $W^\perp$ is a vector subspace of $\mathbb{R}^n$.

i. Let $\vec{w} \in W$. $\vec{0}\cdot\vec{w} = 0 \Rightarrow \vec{0} \in W^\perp$.

ii Let $\vec{v}_1 \in W^\perp$, $\vec{v}_2 \in W^\perp$, $\vec{w} \in W$. By def of $W^\perp$, $\vec{v}_1 \cdot \vec{W} = 0$ and $\vec{v}_2 \cdot \vec{w} = 0$.

$\therefore (\vec{v}_1 + \vec{v}_2)\cdot\vec{w} = \vec{v}_1\cdot\vec{w} + \vec{v}_2\cdot\vec{w} = 0 + 0 = 0$, so $\vec{v}_1 + \vec{v}_2 \in W^\perp$.

iii Let $\vec{v}_1 \in W^\perp$, $c \in \mathbb{R}$, $\vec{w} \in W$. By def of $W^\perp$, $\vec{v}_1\cdot\vec{w} = 0$.

$\therefore (c\vec{v}_1)\cdot\vec{w} = c(\vec{v}_1\cdot\vec{w}) = c\cdot 0 = 0$, so $c\vec{v}_1 \in W^\perp$

i, ii, and iii $\Rightarrow W^\perp$ is a subspace.

2. (3pts) Let $\vec{x}_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $\vec{x}_2 = \begin{bmatrix}5\\0\\1\end{bmatrix}$, and $\vec{w} = \begin{bmatrix}1\\1\\1\end{bmatrix}$. Let W be the two-dimensional subspace of $\mathbb{R}^3$ defined by $W = \text{span}\{\vec{x}_1, \vec{x}_2\}$. Compute the projection of the vector $\vec{w}$ onto W. Do not simplify your answer.

First, find an orthogonal basis for W:

Let $\vec{v}_1 = \vec{x}_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$ $\quad \vec{v}_2 = \vec{x}_2 - \dfrac{\vec{x}_2\cdot\vec{v}_1}{\|\vec{v}_1\|^2}\vec{v}_1 = \begin{bmatrix}5\\0\\1\end{bmatrix} - \dfrac{5}{5}\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}4\\-2\\1\end{bmatrix}$

Now: $\text{proj}_W \vec{w} = \dfrac{\vec{w}\cdot\vec{v}_1}{\|\vec{v}_1\|^2}\vec{v}_1 + \dfrac{\vec{w}\cdot\vec{v}_2}{\|\vec{v}_2\|^2}\vec{v}_2 = \dfrac{3}{5}\begin{bmatrix}1\\2\\0\end{bmatrix} + \dfrac{3}{21}\begin{bmatrix}4\\-2\\1\end{bmatrix}$

3. (EC 2pts) Consider the vector space $\mathbb{P}_4$ (polynomials of degree 4 or less) with the inner product $< p_1, p_2 >= \int_{-1}^{1} p_1(t)p_2(t)dt$. Let $f(t) = t^2$, and $g(t) = t^4$. Compute the projection of the vector g onto the vector f.

$$\text{proj}_{\text{Span}\{t^2\}} t^4 = \frac{\langle t^4, t^2\rangle}{\langle t^2, t^2\rangle} t^2 = \frac{2/7}{2/5} t^2 = \frac{5}{7}t^2$$

$$\langle t^4, t^2\rangle = \int_{-1}^{1} t^4\cdot t^2\, dt = \frac{t^7}{7}\Big|_{-1}^{1} = \frac{2}{7}$$

$$\langle t^2, t^2\rangle = \int_{-1}^{1} t^2\cdot t^2\, dt = \frac{t^5}{5}\Big|_{-1}^{1} = \frac{2}{5}$$