Math 3280 Practice Test 3. Directions: SHOW ALL WORK. Please do not write on the provided Laplace Transform tables.
Indicate clearly any places where you use either the Laplace transform tables or a calculator.

1. (6 pts) From Calculus II, you can determine the solution to $y'' = 1$ by integration. Do this to find the general solution. Identify in this solution y_c (the solution to the corresponding homogeneous differential equation) and y_p, the particular solution.

2. (6 pts) Find one particular solution to $y'' + 2y' + 4y = 3e^{2x}$. (You need not find the general solution.)

3. (6 pts) Find the general solution to the constant coefficient linear homogeneous differential equation which has the following characteristic polynomial: $(r^2+4)(r-4)(r+4)^2$ (obtained by trying a solution of the form $y(x) = e^{rx}$).

4. (6 pts) What differential operator annihilates $5\cos(3x)$? Verify your answer.

5. (6 pts) Find the form of a particular solution to $y'' - y' - 2y = 3e^{2t}$ Do not include extraneous terms and do not evaluate the "undetermined coefficients."

6. (6 pts) What is the Laplace transform of $g(t) = 3te^{2t} - u(t-4)$? (You may use the tables. You need not simplify your answer.)

7. (6 pts) **Use the definition of the Laplace transform** (not the tables) to show that the Laplace transform of e^{3t} is $\dfrac{1}{s-3}$.

8. (6 pts) Define $f(t) = \begin{cases} 3 & 0 \le t < 3 \\ t^2 & 3 \le t < 4 \\ \cos(t) & 4 \le t. \end{cases}$ Use step functions to write $f(t)$ as a single line formula.

9. (6 pts) Solve using the method of Laplace transforms: $y'(t) - 4y(t) = 0,\ y(0) = 2$.

10. (7 pts) Compute the Laplace transform of the solution of the initial value problem:

$$y'' + y' + 2y = 3\cos(4t); \quad y(0) = 0, y'(0) = 3.$$

(Find only $Y(s)$, not $y(t)$.) Write your answer as a polynimial (in s) over a polynomial. You need not simplify your answer.

11. (7 pts) Find the inverse Laplace transform of $F(s) = \frac{3s-5}{s^2+s-12}$.

12. (7 pts) Write the differential equation $y'' - 6y' - 2y = \cos(2t)$ with initial conditions $y(0) = 3, y'(0) = 4$ as an equivalent system of first order differential equations. Write the system in vector form: $\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}$. Write the initial conditions in vector form as well.

13. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$.

    (a) (4 pts) The matrix A has an eigenvalue 2 with corresponding eigenvector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$). Verify this using the definition of an eigenvalue and eigenvector.

    (b) (4 pts) Use part(a) along with the fact that A has a second eigenvalue of -3 with corresponding eigenvector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$) to find the general solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$.

    (c) (4 pts) Use both parts (a) and (b) to determine the solution to $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$ which also satisfies the initial conditions $\mathbf{x}(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$.

14. (7 pts) Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Find any one eigenvalue and correpsonding eigenvector for A.

15. Consider the system of differential equations

$$\dot{x} = 2x - x^2 - xy, \quad \dot{y} = -y + xy$$

    (a) (3 pts) Find any one equilibrium point.

    (b) (3 pts) Compute and sketch in the (x, y) phase plane the two velocity vectors at $(x, y) = (2, 1)$ and $(x, y) = (1, 1/2)$.

16. (Extra credit 6 pts) **Use the definition of the Laplace transform** to show that if $F(s)$ is the Laplace transform of $f(t)$, then the transform of $f'(t)$ is $sF(s) - f(0)$. (You need not provide details why "plugging in at ∞" is zero.)