# Math 3280
# Differential Equations with Linear Algebra Practice Test 2
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1. Consider the differential equation $y'' + 2y' - 3y = 3x - 5$.

   (a) (4 pts) Verify that $f(x) = -x + 1$ is a solution to this differential equation.

   (b) (8 pts) Use this information to help find the general solution to the differential equation.

2. (6 pts) Find the general solution to $y'' + 6y' + 9y = 0$.

3. (8 pts) Show that the three functions $1, x, x^2$ are linearly independent. Work directly from the definition of linear independence/dependence. (That is, do not just compute a Wronskian determinant without indicating why it is being computed.)

4. The three functions $1, x$, and $x^2$ are all solutions to the differential equation $y'''(x) = 0$. (You do not need to verify this.)

   (a) (4 pts) Use this information to write the general solution to $y'''(x) = 0$.

   (b) (4 pts) Find the one solution to $y'''(x) = 0$ along with the initial conditions $y(0) = 1, y'(0) = 2, y''(0) = 3$.

5. (4 pts) Write any system of equations in 5 variables, $x_1, x_2, ..., x_5$ which has a solution set which is a 3-dimensional subspace of $\Re^5$. You can decide how many equations to use.

6. (6 pts) Evaluate the following determinant. Show your work.

$$\begin{vmatrix} 0 & 3 & -1 & 2 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -2 & 0 \end{vmatrix}$$

7. (4 pts) Give a geometric description in words of the set $W = span \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$. Include a statement of the space in which this set lives as well as the description of $W$.

8. (4 pts) Write the vector equation $r \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ in the form $A\mathbf{x} = \mathbf{b}$.

9. (8 pts) Solve the following linear system USING GAUSSIAN ELIMINATION (row reduction to echelon or reduced echelon form). Leave your answers as exact fractions - not calculator approximations.

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

10. (a) (8 pts) Find all solutions to $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Write your answer in vector form.

    (b) (2 pts) What is the dimension of the set of solutions to part (a)?

11. (4 pts) Write down any basis for $\Re^3$ that includes the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Justify briefly. Neither computation nor formal proof is required.

12. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

    (a) (6 pts) Find $A^{-1}$ using the Gauss-Jordan (row reduction) technique.

    (b) (2 pts) Check your answer.

13. ( 6pts) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$. Define a matrix $B$ so that $BA = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 + 2a_1 & a_8 + 2a_2 & a_9 + 2a_3 \end{bmatrix}$

14. (4 pts) (True or False) $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. Justify using the definition of span.

15. (8 pts) Consider the following subset $W$ of $\Re^2$. PROVE that $W$ is a vector subspace of $\Re^2$.

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \Re^2 : x_2 = 0 \right\}$$

16. Extra credit (6 pts) Consider the differential equation $y'' + x^2 y' + y = 0$. Assume that $y_1(x)$ $y_2(x)$ are both solutions to this differential equation. Show that $y_1(x)+y_2(x)$ is also a solution to the same differential equation.