Math 3280: DE+LA. Test 1 partial answers. Prof. Bruce Peckham

1. (a) Both separable and linear
   (b) separable but not linear
2. 3
3. (a) $y(x) = \frac{3}{2}x^2 + \frac{1}{2}$
   (b) $P(s) = 4e^{-5s}$
   (c) $x(t) = te^{-t^3} + Ce^{-t^3}$
   (d) $\frac{y^{-2}}{-2} = \frac{x^4}{4} + \frac{x^3}{3} + x + C$ (implicit solution)
4. (a) $\dot{W} = -10W$
   (b) i. $W(t) = Ce^{-10t}$
   ii. $T(t) = 70 - Ce^{-10t}$ (C could also be replaced with C/10.)
5. Sketch not provided. (All solutions should be parabolas $x^2 + C$).
6. $\phi(1.1) = -1, \phi(1.2) = -0.99$
7. $m = -\frac{1}{3}$ OR $A = 0$.
8. Sketch not provided. Phase line should include dots at equilibria: $P = 0, 1, 3$, and arrows inbetween equilibria. $P(0)$ must be greater than 1 in order for the population to survive. In this case, the population approaches 3. Otherwise, the population dies out.
9. Let $M(t)$ be the number of mathematicians alive at time t.

$$\dot{M} = k_1 M - k_2 M^2 - 100, M(0) = 10,000$$

   k_1 and k_2 are both positive constants.