Name A.K.

Diff. Equations and Lin. Alg.
Math 3280
Quiz 1, Spring 2018
B. Peckham

1. (1 pt) Consider the following differential equation. Is it separable, linear, both or neither? (Do not solve.)

   $y' = ye^x$ Both

2. (1 pt) Is the function $\phi(x) = e^{5x}$ a solution to $y'' - 3y' + y = 3e^{5x}$? Justify your answer.

   If $\phi(x) = e^{5x}$, then $\phi'(x) = 5e^{5x}$, $\phi''(x) = 25e^{5x}$

   Plug in $\phi(x)$ for y: $25e^{5x} - 3\cdot 5e^{5x} + e^{5x} = 11e^{5x} \neq 3e^{5x}$ $\therefore$ No

3. (4 pts) Obtain the general solution to the following differential equation and the solution that satisfies the initial value problem. Show your work and clearly indicate your final answer.

   $$\frac{dr}{dt} = 3r + 2, r(0) = 2.$$

   2 methods: Sep of var or 1st order linear

   Sep of var $\frac{1}{3r+2}\frac{dr}{dt} = 1$

   Int. $\int \frac{1}{3r+2}\frac{dr}{dt}\,dt = \int 1\,dt$

   $\Rightarrow \frac{1}{3}\int \frac{3}{3r+2}\,dr = t + c$

   $\Rightarrow \frac{1}{3}\ln|3r+2| = t+c$ (implicit sln)

   1st order linear method

   $r' - 3r = 2$

   Int. factor e^{-3t}

   $\Rightarrow r'e^{-3t} - 3re^{-3t} = 2e^{-3t}$

   i.e. $(re^{-3t})' = 2e^{-3t}$

   Int: $re^{-3t} = 2\frac{e^{-3t}}{-3} + C$

   $\Rightarrow \boxed{r = -\frac{2}{3} + Ce^{3t}}$

4. (2 pts) Consider the differential equation $\frac{dP}{dt} = (2P+t)^2$. Make the substitution $u = 2P + t$ to eliminate the variable P. Write the new differential equation. Show your work. Do not solve the differential equation. Extra Credit (+1 pt) Is this substitution useful in solving the differential equation? Why?

   $u = 2P+t \Rightarrow \frac{du}{dt} = 2\frac{dP}{dt} + 1 = 2(2P+t)^2 + 1$

   $= 2u^2 + 1$

   i.e. $\boxed{u' = 2u^2+1}$ This is separable. $\therefore$ Sub is useful

Let $T(t)$ be the temperature at time t. T_0 = outdoor temp (constant) $\boxed{\dot{T} = k(T - T_0)}$

5. (2 pts) Write a differential equation for the following heating/cooling model: assume that the rate of change of the temperature inside a house is proportional to the difference between the indoor temperature and the outdoor temperature. Define your variables. Do not solve.

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Diff. Equations and Lin. Alg.
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Quiz 2, Fall 2014

1. (3 pts) Use USING GAUSSIAN ELIMINATION (row reduction operations) on the "augmented matrix" to convert it to row echelon form to find all solutions to the following system. Write your answer in vector form.

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 2 & -2 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \Rightarrow x_3 = t$

$x_2 - x_3 = 1 \Rightarrow x_2 = t + 1$

$x_1 + 2x_3 = 0 \Rightarrow x_1 = -2t$

2. (2 pts) Does the set of solutions to problem 1 form a vector subspace of $\Re^3$? Justify briefly.

NO. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is not a soln! (Or show the sum of 2 solutions is not a solution, Or show 2 times a solution is not a solution)

3. (3 pts) Find a linear combination of the three vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ which equals $\begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}$.

$a \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 5 \\ 1 & 0 & 0 & 3 \end{array}\right] \rightarrow \begin{array}{cccc} 1 & 1 & 1 & -1 \\ 0 & 0 & -1 & 6 \\ 0 & -1 & -1 & 4 \end{array} \rightarrow \begin{array}{cccc} 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & -1 & 6 \end{array} \Rightarrow$ $c = -6$, $b - 6 = -4 \Rightarrow b = 2$, $a + 2 - 6 = -1 \Rightarrow a = 3$

4. (2 pts) If A is a 5×5 matrix, and $\det(A) = 1$, what is $\det(2A)$? Explain briefly.

$\det(2A) = 2^5 \cdot 1 = 32$

5. (2 pts) Assume A and B are both 3×3 invertible matrices with respective inverses A^{-1} and B^{-1}. Show that the inverse of the product AB is $B^{-1}A^{-1}$.

6. (3 pts) Consider the differential equation: $\dot{x}(t) = (3x(t)^2 + t)^2$. Make the substitution $v(t) = 3x(t)^2 + t$ to eliminate $x(t)$. What is the new differential equation in v and t? Is this a useful substitution to solve the original differential equation?

$v = 3x^2 + t \Rightarrow \dot{v} = 6x\dot{x} + 1 = 6x v^2 + 1 = 6\left(\frac{v-t}{3}\right)^{\frac{1}{2}} v^2 + 1$

$x^2 = \frac{v-t}{3} \Rightarrow x = \left(\frac{v-t}{3}\right)^{\frac{1}{2}}$

7. Extra Credit: (3 pts) Give an example of a 2×2 matrix A, and two vectors $\vec{u}$ and $\vec{v}$ in $\Re^2$ such that $A\vec{u} = A\vec{v}$ but $\vec{u} \neq \vec{v}$.

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Diff. Equations and Lin. Alg.
Math 3280
Quiz 3, Spring 2018
B. Peckham

1. (4 pts) Find the general solution to the following differential equation:

$$y'' + 3y' + 2y = e^{5x}$$

Try $y = e^{rx} \rightarrow r^2 + 3r + 2 \Rightarrow (r+1)(r+2) \rightarrow$ basis $\{e^{-x}, e^{-2x}\} \Rightarrow y_c(x) = c_1 e^{-x} + c_2 e^{-2x}$

Try $y_p = Ae^{5x} \Rightarrow y' = 5Ae^{5x},\ y'' = 25Ae^{5x}$

Plug in: $(25A + 3\cdot 5A + 2A)e^{5x} = 1e^{5x}$

$\Rightarrow 42A = 1 \Rightarrow A = \frac{1}{42}$

$\therefore y(x) = y_c + y_p$

$= c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{42} e^{5x}$

2. (3 pts) Write down the form of particular solution for the following differential equations. Do not include "extraneous" terms. Do not evaluate the "undetermined" coefficients.

(a) $y'' + 4y = \cos(x)$ $\quad y_p = A\cos x + B\sin x$ (or just $A\cos x$) (Since there is no y' term)

(b) $y'' + 4y = \cos(2x)$ $\quad y_p = Ax\cos 2x + Bx\sin 2x$ (Since $\sin 2x + \cos 2x$ are sols to $L(y) = 0$.)

3. (4 pts) Find the general solution to the following differential equation:

$$y'' - 2y' + 5y = 0$$

Try $e^{rx} \rightarrow r^2 - 2r + 5 = 0$ or $(r-1)^2 = -4$ or $r = 1 \pm 2i$

$\Rightarrow y_1(x) = e^x \cos 2x,\ y_2(x) = e^x \sin 2x \Rightarrow y(x) = c_1 e^x \cos 2x + c_2 e^x \sin 2x$

4. (2 pts) Extra Credit: Give an example of a linear, constant coefficient homogeneous differential equation that has two solutions: $\cos(x)$, and e^{7x}. (Hint: what order must the differential equation be?) $\cos x$ sln $\Rightarrow r = \pm i$; e^{7x} sln $\rightarrow r = 7$.

$\therefore$ Roots $\pm i, 7$ $\quad \therefore (r-i)(r+i)(r-7) = (r^2+1)(r-7)$

$= r^3 - 7r^2 + r - 7$

$\Rightarrow y^{(3)} - 7y^{(2)} + y' - 7y = 0$ is the d.e.

Name $A.k.$

Math 3280, Spring 2018
Quiz 4a
B. Peckham

1. Consider the initial value problem: $y' + 2y = e^{4t}, y(0) = 3$.

   (a) (4pts) Solve completely using the method of Laplace transforms. Hint: $\frac{3s-11}{(s+2)(s-4)} = \frac{1/6}{s-4} + \frac{17/6}{s+2}$.

   $Y(s)(s+2) = 3 + \frac{1}{s-4}$

   $Y(s) = \frac{3(s-4)+1}{(s+2)(s-4)} = \frac{3s-11}{(s+2)(s-4)} = \frac{1/6}{s-4} + \frac{17/6}{s+2}$

   $\Rightarrow y(t) = \frac{1}{6}e^{4t} + \frac{17}{6}e^{-2t}$

   (b) (4pts) Solve completely by "guessing" e^{rt} for the complementary solution (y_c), and finding y_p by the method of undetermined coefficients.

   $y_c = e^{rt} \Rightarrow y_c' = re^{rt}$

   Plug in: $re^{rt} + 2e^{rt} = 0$

   $\Rightarrow r = -2$

   $\Rightarrow y_c = Ce^{-2t}$

   Try $y_p = Ae^{4t}$

   $\Rightarrow y_p' = 4Ae^{4t}$

   Plug in $4Ae^{4t} + 2Ae^{4t} = 1e^{4t}$

   $\Rightarrow 4A + 2A = 1 \Rightarrow A = \frac{1}{6}$

   $\therefore y_p = \frac{1}{6}e^{4t} \Rightarrow y(t) = y_c + y_p = Ce^{-2t} + \frac{1}{6}e^{4t}$

   $y(0) = 3 \Rightarrow C + \frac{1}{6} = 3 \Rightarrow C = \frac{17}{6}$

   $\therefore y(t) = \frac{17}{6}e^{-2t} + \frac{1}{6}e^{4t}$

2. (2pts) Find the inverse Laplace transform of $e^{2s}\frac{6s+4}{s^2+4}$.

   $F(s) = \frac{6s+4}{s^2+4} = 6\frac{s}{s^2+4} + 2\cdot\frac{2}{s^2+4} \Rightarrow f(t) = 6\cos 2t + 2\sin 2t$

   $\therefore e^{-(-2)s}F(s) \longrightarrow u(t+2)\left(6\cos 2(t+2) + 2\sin 2(t+2)\right)$

3. (2pts) Find the form of a guess for a particular solution to the following differential equation. Do not include any terms that are solutions to the complementary (homogeneous) differential equation. Do not evaluate the undetermined coefficients.

   $y'' + y' - 2y = e^{2x} + e^{-2x}$.

   $y_p = Ae^{2x} + Bxe^{-2x}$

   $r^2 + r - 2 = (r+2)(r-1)$

   $\Rightarrow r = 1, -2$

4. (2pts) EC What is the annihilator of $e^{2x} + e^{-2x}$?

   $(D-2)(D+2) = D^2-4$

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Diff. Equations and Lin. Alg.
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Quiz 5, Spring 2018

1. Condider the differential equation $y' - 3y = 2e^{3x}$.

   (a) (3pts) Find the general solution using the first order linear technique by finding an integrating factor.

$\mu(x) = e^{\int -3dx} = e^{-3x}$. Mult both sides: $y'e^{-3x} - 3ye^{-3x} = 2e^{3x}e^{-3x}$

ie $(ye^{-3x})' = 2$

integrate: $ye^{-3x} = 2x + C$

Solve for y: $\boxed{y = 2xe^{3x} + Ce^{3x}}$

   (b) (3pts) Solve using "guess e^{rx}"for y_c and undetermined coefficients for y_p. Give the general solution.

For y_c: Try $y_c = e^{rx}$. Plug in: $re^{rx} - 3e^{rx} = 0 \Rightarrow r - 3 = 0 \Rightarrow \underline{r = 3}$, $\therefore y_c = Ce^{3x}$.

For y_p: Try $y_p = Ae^{3x}$. Since e^{3x} is part of y_c, "adjust" y_p to be Axe^{3x}.

Plug in: $y_p' = A(x\,3e^{3x} + 1\,e^{3x})$ So $y_p' - 3y_p = (A\,x\,3e^{3x} + Ae^{3x}) - 3Axe^{3x}$

$= Ae^{3x} = 2e^{3x} \Rightarrow A = 2$, So $y_p = 2xe^{3x}$

$\therefore y = y_c + y_p = \underline{Ce^{3x} + 2xe^{3x}}$

2. (4 pts) Use USING GAUSSIAN ELIMINATION (row reduction operations) on the "augmented matrix" to convert it to row echelon form to find all solutions to the following system. Write your answer in vector form, and find a basis for the set of solutions.

   $3x + 5y + z - w = 0$, $x + y - z + 5w = 0$.

$$\left[\begin{array}{cccc:c} 3 & 5 & 1 & -1 & 0 \\ 1 & 1 & -1 & 5 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc:c} 1 & 1 & -1 & 5 & 0 \\ 3 & 5 & 1 & -1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc:c} 1 & 1 & -1 & 5 & 0 \\ 0 & 2 & 4 & -16 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc:c} 1 & 1 & -1 & 5 & 0 \\ 0 & 1 & 2 & -8 & 0 \end{array}\right]$$

z, w free: Let $\underline{z = t}$, $\underline{w = s}$. 2^{nd} eq: $y + 2z - 8w = 0 \Leftrightarrow y = -2z + 8w = \underline{-2t + 8s}$

1^{st} equ: $x + y - z + 5w = 0 \Leftrightarrow x = -y + z - 5w = -(-2t + 8s) + t - 5s = \underline{3t - 13s}$

Vector form: $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = t\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -13 \\ 8 \\ 0 \\ 1 \end{bmatrix}$

3. (5pts) Let $S = \{\vec{x} : A\vec{x} = \vec{0}\}$.

   - Assume A is a 5×7 matrix. For what value of k is S a subspace of $\Re^k$? 7
   - Prove that S a subspace. (You need not prove the "inherited" properties, only the two closure properties.)

A. Closure under vector addition

Assume $\vec{u}, \vec{v} \in S$. This means $A\vec{u} = \vec{0}$ and $A\vec{v} = \vec{0}$ (1)

$\therefore A(\vec{u} + \vec{v}) = A\vec{u} + A\vec{v}$ (by properties of matrix multiplication)

$= \vec{0} + \vec{0}$ by (1)

$= \vec{0}$ $\therefore \vec{u} + \vec{v} \in S$.

B. Closure under scalar multiplication

Let $\vec{u} \in S$, $c \in \mathbb{R}$.

$\vec{u} \in S \Rightarrow A\vec{u} = \vec{0}$ (2)

$\therefore A(c\vec{u}) = c(A\vec{u})$ by properties of matrix mult.

$= c(A\vec{u})$ by (2)

$= c\,\vec{0} = \vec{0}$ $\therefore c\vec{u} \in S$

A and B $\Rightarrow S$ is a vector subspace of $\mathbb{R}^7$.