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Diff. Equations and Lin. Alg.
Math 3280 , B. Peckham
Quiz 2, Fall 2014

1. (3 pts) Use USING GAUSSIAN ELIMINATION (row reduction operations) on the "augmented matrix" to convert it to row echelon form to find all solutions to the following system. Write your answer in vector form.

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

Vector form: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4-2t \\ t-\frac{3}{2} \\ t \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{3}{2} \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 1 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 2 & -2 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & -\frac{3}{2} \end{array}\right]$

or $x_3 = t$

$x_2 - x_3 = -\frac{3}{2} \Rightarrow x_2 = t - \frac{3}{2}$

$x_1 + 2x_3 = 4 \Rightarrow x_1 = 4 - 2x_3 = 4 - 2t$

2. (2 pts) Does the set of solutions to problem 1 form a vector subspace of $\Re^3$? Justify briefly.

No. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is not a sln. (or show the set of solutions is not closed under addition, or " " " " " " scalar multiplication.)

3. (3 pts) Find a linear combination of the three vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ which equals $\begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$.

$a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow c = 2$, $b + c = 5 \Rightarrow b = 3$, $a + b + c = -1 \Rightarrow a = -6$

So $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} = -6\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

4. (2 pts) If A is a 4×4 matrix, and $\det(A) = 1$, what is $\det(2A)$? Explain briefly.

$\det(2A) = 2^4 \cdot \det A = 2^4 \cdot 1 = 16$

Ex: $\left|\begin{pmatrix} 1 & 2 & 5 & 8 \\ 1 & 0 & 3 & \\ 1 & 4 & & \\ 1 & & & \end{pmatrix}\right| = 1$, but $\left|\begin{pmatrix} 2 & 4 & 10 & 16 \\ 2 & 0 & 6 & \\ 2 & 8 & & \\ 2 & & & \end{pmatrix}\right| = 16$.

5. (2 pts) Assume A and B are both 3×3 invertible matrices with respective inverses A^{-1} and B^{-1}. Show that the inverse of the product AB is $B^{-1}A^{-1}$.

$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$

$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I$

$\therefore B^{-1}A^{-1}$ is the inverse of AB.

6. (3 pts) Consider the differential equation: $\dot{x}(t) = (3x(t)^3 + t)^2$. Make the substitution $v(t) = 3x(t)^3 + t$ to eliminate $x(t)$. What is the new differential equation in v and t? Is this a useful substitution to solve the original differential equation? No

$v = 3x^3 + t \Rightarrow \dot{v} = 9x^2 \cdot \dot{x} + 1 = 9x^2(3x^3 + t)^2 = 9x^2 v^2 = 9\left(\frac{v-t}{3}\right)^{2/3} v^2$

$\Rightarrow x^3 = \frac{v-t}{3} \Rightarrow x^2 = (x^3)^{2/3} = \left(\frac{v-t}{3}\right)^{2/3}$

ie, $\dot{v} = 9\left(\frac{v-t}{3}\right)^{2/3} v^2$

7. Extra Credit: (3 pts) Give an example of a 2×2 matrix A, and two vectors $\vec{u}$ and $\vec{v}$ in $\Re^2$ such that $A\vec{u} = A\vec{v}$ but $\vec{u} \neq \vec{v}$.

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ but $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Many other correct examples.