Name____
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Math 1596: Honors Calculus I
Test 3
Prof. Bruce Peckham

Directions:

- Calculators are not expected to be helpful, but may be used. Exception: Calculators capable of symbolic manipulation such as the TI 89 are NOT allowed. **You must indicate clearly any step for which you use your calculator.** Certain problems as for "exact answers." This means that (approximate) answers obtained with a calculator will not be given full credit.
- Show all work involved in reaching your answer. **The right answer without appropriate justification will not necessarily receive full credit, or even any credit. Common sense should prevail.**
- Clearly mark your answers.
- Read each question carefully.
- Label all diagrams.
- Leave all numeric answers in exact form (not a decimal approximation from a calculator). If you cannot answer any one part, but that part is needed for subsequent parts, describe how you would obtain answers to the subsequent parts if you did know the answer to the previous part(s).
- Use the back of the test pages for scratch work.
- Make no mistakes. :)

Page	Score	Out of
2		30
3		23
4		18
5		17
6		12
Total		100

Unless otherwise noted, each part of each problem is worth 6 points.

1. Consider the definite integral $\int_0^2 x^2 dx$.

   (a) (6pts) Approximate this integral with a Riemann sum using 4 equal length subintervals and "right-hand endpoint" function evaluations.

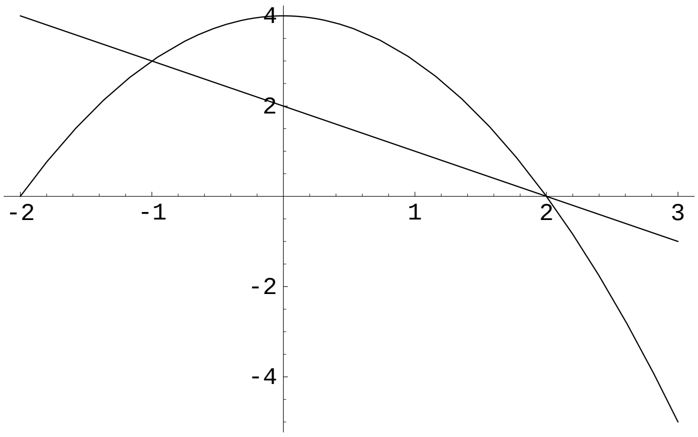
   (b) (6pts) Approximate this integral using 4 equal length subintervals and the trapezoid method.

   (c) (6pts) Approximate this integral with a Riemann sum using n equal length subintervals and "right-hand endpoint" function evaluations.

   (d) (6pts) Evaluate the exact value of the integral using the the Riemann sums in the previous step and letting the number n of intervals go to infinity. You may use the formula: $\sum_1^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

   (e) (6pts) Evaluate the exact value of the integral using the Fundamental Theorem of Calculus (Part 1).

2. (5 pts) Below are the graphs of $f(x)$, the curved graph, and $g(x)$, the straight line. Is $\int_{-2}^{0}(f(x) - g(x))dx$ positive or negative? Justify briefly.

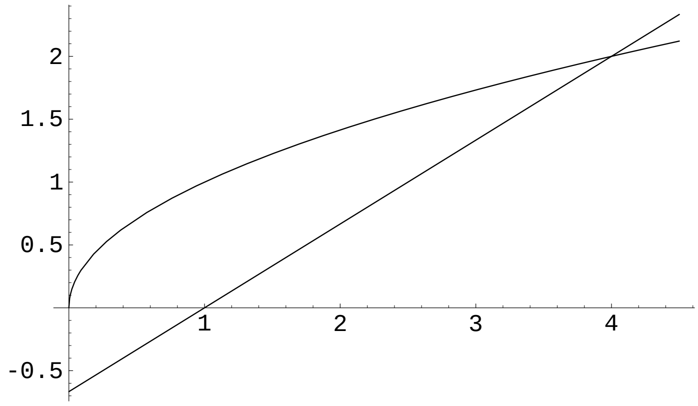


3. (6 pts) Use a u-substitution to evaluate $\int_0^2 \frac{2x^3}{(3+x^4)^3}dx$. You need not simplify your answer.

4. (6 pts) Use integration by parts to find $\int x \sin(5x)dx$.

5. (6 pts) Consider the following improper integral: $\int_0^8 x^{-\frac{1}{3}}dx$ Use the definition of the improper integral to show whether the integral converges or diverges. If convergent, determine to what it converges.

6. Below are the graphs of $y = \sqrt{x}$ and $y = \frac{2}{3}x - \frac{2}{3}$. Call R tregion which is both enclosed between the two curves AND in the first quadrant AND between $x = 0$ and $x = 4$. Call S the solid obtained by revolving R around the x axis.



Set up an integral (or integrals) for the following:

(a) (6pts) the volume of S by integrating with respect to x.

(b) (6pts) the volume of S by integrating with respect to y.

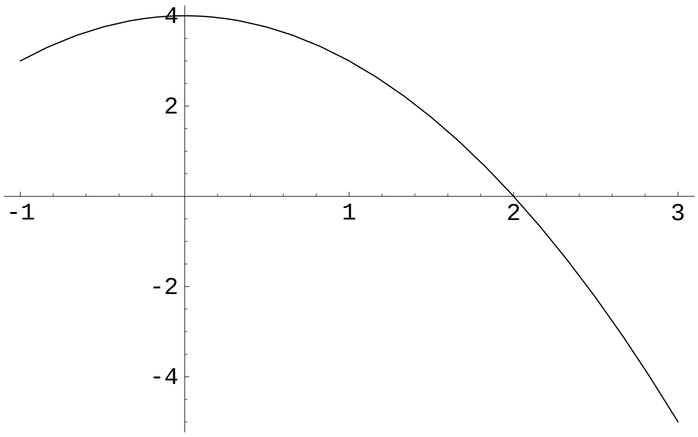
(c) (6pts) the surface area of S. (All parts of the surfaces!)

7. Let $F(x) = \int_1^x \frac{1}{1+t^3} dt$.

(a) (5 pts) What is $F'(2)$? Explain briefly.

(b) (6 pts) Define $g(x) = F(\frac{1}{x})$. What is $g'(x)$?

8. (6 pts) Below is the graph of $y = 4 - x^2$. Call R the region under the parabola AND in the first quadrant. Set up an integral(s) to determine $\overline{y}$, the y-coordinate of the center of mass of R. You need not evaluate the integral(s).



9. (6 pts) Assume f is a continuous function. Let $F(x) = \int_0^x f(t)dt$. Show that $F'(x) = f(x)$.

10. (6 pts) Let $\{a = x_0, x_1, ..., x_n\}$ be a partition of $[a, b]$. Assume we are approximating the arclength of the curve $y = f(x)$ between $(a, f(a))$ and $(b, f(b))$ by adding up all lengths of the straight lines corresponding to the partition points. That is, we add up the lengths of the straight lines connecting $(x_{i-1}, f(x_{i-1}))$ and $(x_i, f(x_i))$ for $i = 1, 2, ..., n$. Show that the contribution to the arclength approximation Δs_i in the i^{th} subinterval is $\Delta s_i = \sqrt{1 + (\frac{\Delta y_i}{\Delta x_i})^2}\Delta x_i$. (State what Δx_i and Δy_i are in terms of the partition points and f. A sketch might be useful.)