Name_____ Math 1596: Honors Calculus I
ID # _____ Test 2
Signature_____ Prof. Bruce Peckham

Directions:

- Calculators are not expected to be helpful, but may be used. Exception: Calculators capable of symbolic manipulation such as the TI 89 are NOT allowed. **You must indicate clearly any step for which you use your calculator.** Certain problems as for "exact answers." This means that (approximate) answers obtained with a calculator will not be given full credit.
- Show all work involved in reaching your answer. **The right answer without appropriate justification will not necessarily receive full credit, or even any credit. Common sense should prevail.**
- Clearly mark your answers.
- Read each question carefully.
- Label all diagrams.
- Leave all numeric answers in exact form (not a decimal approximation from a calculator). If you cannot answer any one part, but that part is needed for subsequent parts, describe how you would obtain answers to the subsequent parts if you did know the answer to the previous part(s).
- Use the back of the test pages for scratch work.
- Make no mistakes. :)

Page	Score	Out of
2		37
3		31
4		21
5		11
Total		100

Unless otherwise noted, each part of each problem is worth 6 points.

1. (6 pts) Compute the derivative of $f(x) = \dfrac{\sin(x)}{\cos(x)}$.

2. (6 pts) Compute the derivative of $f(x) = \sin\sqrt{x^2 - x^3\cos(x)}$.

3. Find the following antiderivatives:

   (a) (6 pts) $\displaystyle\int (x^3 + 4 + \cos(2x))dx$

   (b) (5 pts) $\displaystyle\int \frac{2x}{(1+x^2)^2}dx$ Hint: Think of how differentiation of some expression using the chain rule might give you back $\frac{2x}{(1+x^2)^2}$.

4. (6 pts) Use implicit differentiation to find $y'(x)$ at $(x, y) = (4, 3)$ if $y(x)$ is implicitly defined by $\sqrt{x} + xy^2 = x + 34$.

5. (8 pts) Use the linear approximation to $f(x) = \sqrt{x}$ near $x = 25$ to approximate $\sqrt{26}$.

6. (5 pts) True or False. If the graph of a function f changes concavity at $x = 3$, and $f''(3)$ exists, then $f''(3) = 0$. Explain briefly.

7. (6 pts) A circular balloon is being filled with air at the rate of 3 cubic feet per minute. At what rate is the radius of the balloon increasing when the radius is 2 feet? (Recall the volume of a sphere is $V = \frac{4}{3}\pi r^3$.) You need not simplify your answer.

8. The average cost per item manufactured in a certain process is given by

$$A(x) = \frac{.001x^2 + 2x + 1000}{x}$$

   (a) (2 pts) What is a reasonable choice of domain for x?

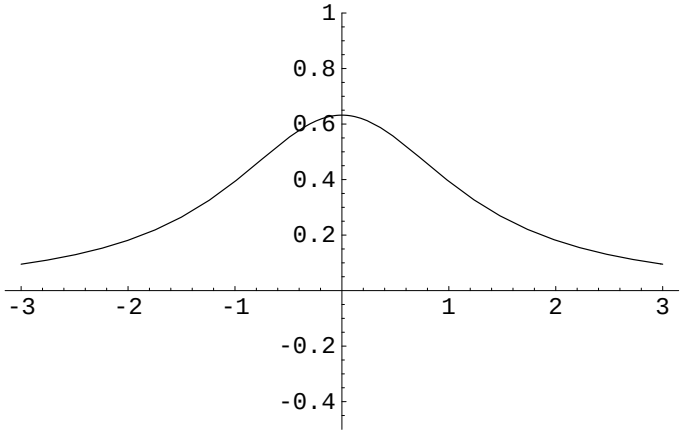
   (b) (8 pts) At what value of x does A assume a global minimum for your choice of domain? Justify.

   (c) (2 pts) What is the value of the global minimum of A? You need not simplify your answer.

9. (8 pts) Find the point on the line $y = \frac{1}{2}x + 1$ that is closest to the point$(1, 0)$. Make a sketch.

10. (6 pts) Assume the national debt is given by $D(t)$. Sketch a plausable graph of D for times $t \in [0, 10]$ which has the debt increasing for all t, but a decreasing rate of increase.

11. (8 pts) Given the following sketch of the graph of f, sketch and label on the same axes the graphs of $f'(x)$ and $f''(x)$.



12. (7 pts) Explain how the Mean Value theorem can be used to guarantee that a function whose first derivatve is always negative can have at most one point at which the original function equals zero.

13. Suppose you are trying to use induction to show that for all positive integers n,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) (3 pts) Establish a base case for $n = 2$.

(b) (8 pts) Prove the inductive step.

(c) (2 pts) State what the base case and the inductive step together prove.

14. Extra credit (5 pts) Prove $\lim_{h \to 0^+} \frac{\sin(h)}{h} = 1$