Name_____ Math 1596: Honors Calculus I
ID # _____ Test 1
Signature_____ Prof. Bruce Peckham

Directions:

- Calculators are not expected to be helpful, but may be used. Exception: Calculators capable of symbolic manipulation such as the TI 89 are NOT allowed. **You must indicate clearly any step for which you use your calculator.** Certain problems as for "exact answers." This means that (approximate) answers obtained with a calculator will not be given full credit.
- Show all work involved in reaching your answer. **The right answer without appropriate justification will not necessarily receive full credit, or even any credit. Common sense should prevail.**
- Clearly mark your answers.
- Read each question carefully.
- Label all diagrams.
- Leave all numeric answers in exact form (not a decimal approximation from a calculator). If you cannot answer any one part, but that part is needed for subsequent parts, describe how you would obtain answers to the subsequent parts if you did know the answer to the previous part(s).
- Use the back of the test pages for scratch work.
- Make no mistakes. :)

Page	Score	Out of
2		42
3		27
4		16
5		15
Total		100

Unless otherwise noted, each part of each problem is worth 5 points.

1. Approximate the arclength of the curve $y = 4 - 4x^2$ that lies in the first quadrant. Use 2 line segments (with endpoints on the graph and x-values of $0, 0.5, 1$). Include a sketch of the graph and the approximating line segments.

2. Evaluate the following limits. Answer with a number, $+\infty$, $-\infty$, or does not exist. Justify very briefly.

   (a) $\lim_{x \to 3} 3x + 5$

   (b) $\lim_{x \to 3^-} \dfrac{5x}{x - 3}$

   (c) $\lim_{x \to 0^+} x \sin(\dfrac{1}{x})$

   (d) $\lim_{x \to \infty} \dfrac{3x^2 + 2x + 1}{x^2 - 3}$

3. Give the formal ϵ-δ definition of what it means to say $\lim_{x \to 2^+} f(x) = 5$.

4. TRUE or FALSE. 3pts each.

   (a) The limit of a product is the product of the limits.

   (b) The derivative of a product is the product of the derivatives.

   (c) $\dfrac{d}{dx}[f(g(x))] = \dfrac{d}{dx}[g(f(x))]$

   (d) If $\lim_{x \to 3^-} f(x)$ does not exit, then $\lim_{x \to 3} f(x)$ cannot exist.

5. Compute the derivatives of the following functions:.

   (a) $\sqrt{3x}(x^2+1)^5$

   (b) $\dfrac{x^2+3}{5x^4-2x^3+1}$

   (c) $\dfrac{(x+1)\sqrt{x}}{(x^{20}+5x^3-3)^{10}}$

6. Suppose the position of a car along a straight highway is given by $d(t) = t + t^{1/3}$.

   (a) (3pts) What is the car's average velocity of the car between times $t = 0$ and $t = 8$?

   (b) (3pts) What is the car's instantaneous velocity at $t = 8$?

7. (6pts) Give an example of a single rational function $f(x) = \dfrac{p(x)}{q(x)}$ with both of the following properties.

   (a) (3pts) $f(x)$ has a slant asymptote of $y = -2x + 3$

   (b) (3pts) $\lim\limits_{x \to 1^-} f(x) = +\infty$

8. (6pts) Sketch a graph of a function f with all the following properties:

   (a) The domain of f is $[0, 5]$.

   (b) $0 \leq f(x) \leq 3$.

   (c) $\lim_{x \to 1^-} f(x) = 2$.

   (d) f has a jump discontinuity at $x = 1$.

   (e) f has a removeable discontinuity at $x = 2$.

   (f) f has a discontinuity at $x = 3$ that is neither a jump discontinuity nor removeable.

   (g) f is continuous at all other points in its domain.

9. (10 pts) Given the following sketch of the graph of f, sketch and label on the same axes the graphs of $f(1/x)$ and $f'(x)$.

10. Compute the derivative of $f(x) = x^2 + 3x$ directly from the definition of derivative.

11. Prove the following directly from the definition of derivative: if $f(x) = p(x)q(x)$, then $f'(x) = p(x)q'(x) + p'(x)q(x)$.

12. Let $f(x) = 2x - 1$. Show directly from the ϵ-δ definition of limit that $\lim_{x \to 2} f(x)$ exists.